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An empirical-mathematical modelling approach to upper secondary physics

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Abstract

In this paper we describe a teaching approach focusing on modelling in physics, emphasizing scientific reasoning based on empirical data and using the notion of multiple representations of physical phenomena as a framework. We describe modelling activities from a project (PHYS 21) and relate some experiences from implementation of the modelling approach in Norwegian upper secondary physics classrooms.

Background

Models in physics and physics education

It may be argued that physics, more than the other sciences, tends to use modelling as a research tool. Within many branches of the physical sciences today, research is essentially about developing and improving *models* (usually formulated in mathematical language) describing phenomena such as climate, the atomic nucleus, or the universe (Chonacky 2006, Gilbert 2004, Winsberg 1999). *Models* and *modelling* consequently receive increasing attention from the science education community as important components of a contemporary science education (e.g. Gilbert 2004, Gilbert and Boulter 2000, GIREP 2006, Greca and Moreira 2002), both because they reflect the nature of physics and because modelling activities are considered useful for learning physics concepts and processes. For instance, Hestenes and his group have developed 'The Modelling Method of High School Physics Instruction' (see Hestenes 1987, Hestenes 1996, Wells *et al* 1995), which is said to correct many weaknesses of the traditional lecture–demonstration method, including fragmentation of knowledge, student passivity, and persistence of naïve beliefs about the physical world. This group emphasizes the similarities between modelling as a cognitive process (as a tool in the individual's learning of physics) and as an activity in physics research.

Another approach is VnR (variables and relationship), which is a computer-based dynamic modelling tool, where the aim is to focus on the patterns in the mathematics rather than calculations (Lawrence 2004). Vitor Duarte Teodoro (2002) has developed Modellus, which is an educational software that makes it possible for students to investigate science phenomena through model building and to use mathematics to create and explore models interactively. Students should be able to start making models from identification of variables, interpret equations, and make visual

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representations without advanced programming. Modellus activities show how mathematics and physics have a unity that is very difficult to see with traditional approaches (see also Teodoro 2004).

The consequences of a 'modelling view of physics' for physics teaching would be that physics education should give students a view of the nature of physics as a modelling enterprise; and physics education should train students to become competent modellers and interpreters of models.

To become competent modellers, students need practice in performing reasoning processes. Scientific reasoning, particularly reasoning on the basis of empirical evidence, has proven difficult for physics students. For instance, Leach (1999) observed that many students are not able to evaluate the logical implication of data for knowledge claims. He pointed out that many students struggle with the co-ordination of theory and empirical evidence, and he suggested that science curricula should include 'the model-like nature of much scientific knowledge, the ways in which predictions are generated and observations are evaluated in terms of stated theories, and the ways in which public scientific knowledge is validated through both social and empirical processes' (pp 803–804). Thus an important component in a modelling approach to physics education would be to give students an understanding of *reasoning* as an essential mediator between experimental observations and theory/model, strengthening the connection between experimental and conceptual representations (see below).

Multiple representations of physical phenomena

Physics has a long tradition for being regarded as a particularly difficult school subject (Angell *et al* 2004, Carlone 2003, Osborne and Collins 2001). Dolin (2002), with basis in Roth (1995), has suggested that physics appears difficult because it requires students to cope with a range of different representations (experiments, graphs, verbal descriptions, formulae, pictures/diagrams) and to manage the translations between these (Dufour-Janvier *et al* 1987). In a project (PHYS 21) in Norwegian upper secondary schools, we have applied a conceptualization of working with physics in terms of *multiple representations*.

The phenomenon 'free fall' may serve as an example: dropping a certain object from a certain height to observe or measure its motion is a simple *experimental* representation of the phenomenon; the velocity may be represented *graphically* as a function of time, and a sketch of a falling object (perhaps with force or velocity vectors) is a *pictorial* representation. Defining change in velocity with time as acceleration is a *conceptual* representation, and the formula $v = v_0 + at$, which in this case reduces to $v = gt$, is one of the *mathematical* representations.

Understanding the physical description of the phenomenon involves learning to simultaneously apply and translate between the various representations, and to refine one's mastery of each representation, for instance by acquiring the appropriate physical concepts and terminology and mathematical 'tools' such as differentiation and integration. For example, the ability to experience a phenomenon and instantly 'see' a graph of and a mathematical relationship between relevant variables—the 'simultaneous' application of and smooth interchange between the forms of representation—is an ability which distinguishes the trained physicist or physics student from the novice.

In this conceptualization, working with physics always involves working with representations of a physical phenomenon. According to Prain and Waldrip (2006), a focus on students' understanding of multiple representations may contribute to effective science learning by catering for students' individual learning needs and preferences and promoting students' active engagement with ideas and evidence.

Dolin (2002) and De Lozano and Cardenas (2002) hold that the 'translation' from physical situations to the formalized language of mathematics represents a particular challenge for physics students. Erickson (2006), in line with our own experiences, found that students solving practical tasks tended to work in either a mathematical mode or a physical mode. A similar observation was made by Taber (2006). The students, for example, easily identified the 'slope' and the 'interception' in linear equations when being in a mathematical mode, but found it difficult to identify the roles of the corresponding constants in a physics formula. Thus, experimental representations and mathematical representations and the relation between these have been especially emphasized in PHYS 21. An increased focus on empirical-mathematical modelling in physics may be expected to give students the training they need to 'interpret' a physical situation in terms of mathematical relationships.

An empirical-mathematical modelling approach to physics instruction

As a response to the challenges and considerations presented above, we have employed an *empirical-mathematical modelling* approach, by which we mean physics teaching emphasizing

- activities in which students employ multiple representations of physical phenomena to conduct experiments and construct and evaluate mathematical models of the phenomena; and
- core messages that physics is a collection of 'models of natural phenomena', often expressed in a mathematical language, and that 'doing physics' means working with models in a wide sense of the term.

We believe that working with multiple representations within a modelling approach may help students to see relations between different representations and to use this actively in their learning process. This approach was implemented in project PHYS 21, which will be presented next.

The PHYS 21 project

In order to try out these ideas in the classroom, we launched project PHYS 21 in Norwegian upper secondary schools with the aim of combining an increased focus on representations of physical phenomena with activities and teaching emphasis on models and modelling. The project took place over a period of three years (2003–2006), and about 20 physics teachers participated in the initial phases of the project, whereas 6 schools, 13 teachers and 289 students took part during the last (full implementation) year.

A teacher booklet introduced the view of physics applied in the project, aspects of scientific method and scientific reasoning, examples of scientific models and the modelling process, and suggestions for student modelling activities. A similar booklet was produced for students.

The curriculum itself was an adaptation of the ordinary national curriculum, with the exception that one out of eight stated attainment targets (thermophysics) was taken away and replaced with *modelling*. However, the idea was not to teach modelling as a separate topic, but as a line throughout the course. Teachers who participated in PHYS 21 were introduced to a rationale for physics emphasizing teaching *about* modelling, as well as teaching students *to do* modelling. Emphasis was put on making the various representations (and the transitions between them) clear to students and helping them develop a perspective on their own understanding and learning and possibly refine their learning strategies in physics. The relationship between mathematics and physics was emphasized, and there was a focus on scientific reasoning related to experimental results, particularly by proposing hypotheses and testing them out experimentally. One topic in particular, *forces and motion*, was taught with a modelling approach. In other topics teachers were asked to implement modelling perspectives as a frame for their teaching and include laboratory tasks training students' modelling skills.

Modelling activities in PHYS 21

In this section we present some of the modelling activities used in the PHYS 21 classrooms.

Bending the ruler

This introductory exercise was designed to introduce the empirical-mathematical modelling idea to the students in a very simple experiment requiring little equipment and no specialized physics knowledge. Students were invited to make a model of the displacement of the tip of a clamped ruler as a function of load (figure 1). The load could be different numbers of identical objects like knives or spoons. That way, the load would be measured in 'number of objects', and there was no need for introducing a unit for force at this stage. The experiment yielded a roughly linear model.

Elongation of jelly babies

Also related to elasticity, but somewhat different in nature, was the exercise in which students were to find a relation between the force and the elongation on stretched jelly babies (elastic jelly sweets). They did measurements (figure 2),

Figure 1. Measuring the displacement of the tip of a clamped ruler as a function of load.

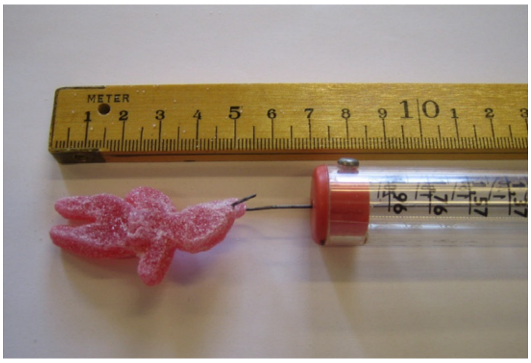


Figure 2. The elastic jelly sweet is stretched and the force measured with a spring scale.

plotted data points in a diagram, suggested a 'best fit line', and from this constructed a model in the form of a mathematical expression (typically a linear relationship valid within certain limits, corresponding to Hooke's law) (figure 3).

The elastic properties of the jelly change when the jelly baby is stretched close to its breaking point. Also, the dependence of elasticity on colouring agent may be studied, since it appears that jelly babies of different colour have different elastic properties.

Finally, a jelly baby that had already been stretched close to its breaking point gave a different slope of the graph when a second elongation was performed. This modelling exercise provides excellent opportunities to discuss such issues as a model's domain of validity, choice of experimental conditions, experimental uncertainty and sources of error, and interpretations of graphs in terms of mathematical expressions and physical quantities.

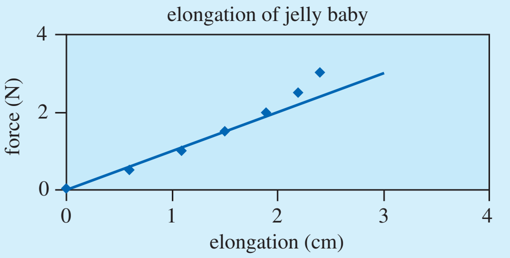


Figure 3. Plot of the force as a function of elongation for a stretched jelly baby.

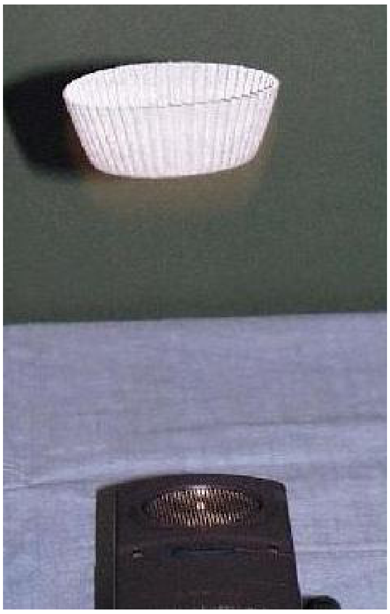


Figure 4. The fall of muffin cups may be studied by using a data logger with a motion sensor.

Air resistance on falling muffin cups

In this exercise, we introduce some more complexity, using second-order equations and a data logger. The task was to investigate how air resistance depends on speed when paper muffin cups are falling (Angell and Ekern 1999). Paper muffin cups fall nicely and evenly when dropped, and they very soon reach terminal velocity. Their motion may be recorded using a data logger (figure 4). 2–5 cups may be placed inside one another, forming an object with the same

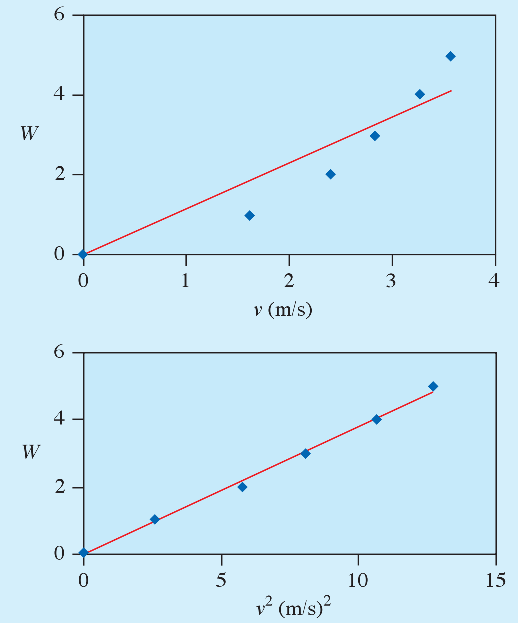


Figure 5. Plots of weight (measured in number of muffin cups) as a function of terminal speed (top) and terminal speed squared (bottom), respectively, allow students to conclude that hypothesis 2 is strengthened: air resistance on falling muffin cups is proportional to the terminal speed squared.

surface but with 2–5 times the weight of one. This investigation may well be performed as a hypothesis testing exercise. Hypothesis 1 could be that air resistance is proportional to the terminal speed; hypothesis 2 that it is proportional to the terminal speed squared. Simple analysis shows that hypothesis 1 implies a linear relationship between the weight of the falling muffin cups and speed; hypothesis 2 a linear relationship between weight and speed squared (figure 5).

The force between two bar magnets

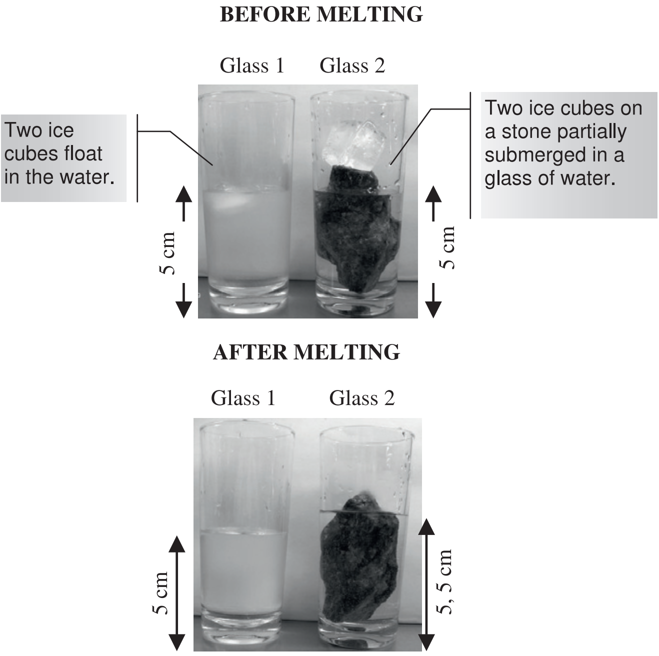
This exercise only requires a simple equipment, but demands attention to experimental error. Furthermore, it involves more complex mathematical relationships, and there is no obvious 'right answer' (in the form of a model). Students investigated the force as a function of distance between the poles of two bar magnets. One magnet was clamped in a vertical position (which was varied during the experiment); the other was placed on a sensitive electronic scale indicating the repulsive force between the two magnetic poles (for practical reasons, the experiment worked better with repulsive than with attractive forces) (figure 6). Interestingly, the different groups of students got quite different results in this exercise, but most of them found a $1/x^n$ dependence when n was between 1 and 2, with x being the distance between the magnets. It should be remarked that the result of this investigation depends quite heavily on the type of magnets used and the geometry. For instance, short 'squat' magnets with larger area of the poles and shorter distance between the poles gives a relationship of the above form but with n of about 3–4 (Gayetsky and Caylor 2007).

Figure 6. The repulsive force between the poles of two bar magnets may be measured by placing one magnet on a sensitive scale and carefully varying the distance to the magnet above it.

Assessment of students' modelling competency

As part of the PHYS 21 project we have also worked with how students' mathematical modelling competency may be assessed. We have suggested and trialled a paper-and-pencil test where 'mathematical modelling competency' is operationalized as the ability to reason scientifically and to see the connection between multiple representations of a phenomenon. Detailed accounts of this assessment instrument and outcomes when using it are presented in Guttersrud (2007).

In the example from the test presented in figure 7, question (1) requires translation between experimental and mathematical representation and



Question 1)

Assume that the ice is melting with a constant rate. Which mathematical expression describes the water level (y) in glass 1 and glass 2 while the ice melts?

A Glass 1: $y = b$, glass 2: $y = ax + b$
B Glass 1: $y = ax + b$, glass 2: $y = b$
C Glass 1: $y = b$, glass 2: $y = ax$
D Glass 1: $y = ax$, glass 2: $y = b$

Question 2)

What does the x in the expressions in the previous question refers to?

A The melting speed of the ice
B The original water level in the glass
C The temperature of the water in the glass
D The time from the ice began to melt

Figure 7. An example of problems designed to measure students' mathematical modelling competency (see Guttersrud (2007)).

question (2) between conceptual and mathematical representation. Both questions require understanding of how to describe a physical situation with general mathematical expressions. Of the almost 500 students (both PHYS 21 project students and regular upper secondary physics students) who completed the test, 67% gave the correct response to the first question and 37% to the second. The two most 'popular' distractors A and B in question 2 refer to the constants a and b in the general mathematical expression given, while distractor C refers to global warming, the underlying concept of the items. Despite a new context and new parameters, students are assumed to be capable to solve the problem in question 1 in view of prior experience with analogous situations. Question 2 challenges students' analytic abilities. Both questions discriminated clearly between the competent and the less competent students.

Experiences and recommendations

Classroom implementation of the PHYS 21 modelling approach

Classroom observations and results from teacher and student questionnaires and interviews indicate that although implementation of the PHYS 21 approach varied quite widely among project classrooms, there was a certain shift in emphasis such that project students reported more emphasis on representations and teachers reported that project participation had changed their teaching practice 'to some extent' and that the 'modelling idea' was prominent in their teaching during 15%–30% of classroom time. Most teachers had applied the modelling approach when teaching mechanics, but they found it difficult to continue in 'modelling mode' in their teaching of other topics. It was not easy for them to see how to implement modelling within other topics such as electricity or light and waves. McDermott *et al* (2000) pointed out that standard physics courses at universities do not provide prospective teachers with a good preparation for designing modelling exercises and working with open-ended investigations in the classroom.

Although project teachers were in general committed to the project and mainly followed project guidelines, the project also illustrates clearly how curriculum ideas are interpreted and adapted to individual teachers' 'identities' and personal backgrounds. Each teacher's specific interests 'merged' with the modelling ideas and led to particular foci. The new curriculum ideas were adapted to teachers' ways of doing and reflecting on teaching, and most teachers found a place for modelling in their personal rationale for teaching physics.

In PHYS 21, we argued for modelling from two main perspectives. The first concerned the nature of physics as models of reality and the task of physicists to construct and apply models, and this perspective was seen as implying that students should learn about science as models and that they should learn 'the tools of the modelling trade'. The second perspective concerned modelling as a powerful tool in the teaching and learning of physics. Classroom observations and teacher interviews show clearly that the last dimension—modelling as a method to teach physics content—was found most attractive by the teachers and most closely matched the teachers' ideas for renewing physics.

Some teachers in PHYS 21 reported difficulties in getting students to adopt the way of thinking and working with physics employed in the project, claiming that students had 'a quite clear expectation of what they are going to learn and how they are going to learn it'. It has been documented before (e.g. Angell *et al* 2004) that students have certain expectations concerning how 'proper physics teaching' is conducted. These expectations are often influenced both by school culture (and school physics culture in particular (Carlone 2003)), but also by parents, peers, etc (Geelan 1997). These strong expectations are related to student beliefs about the nature of the subject matter and about appropriate teaching and learning strategies when working with physics. This issue illustrates the need for a stronger and more explicit focus on developing students' learning strategies and their meta-understanding of physics and physics learning (see below).

Students' outcomes of the PHYS 21 approach

As already mentioned, implementation of the PHYS 21 approach varied widely among classrooms, and our project design did not allow for controlled investigation of the performance of project students in relation to that of regular students. However, questionnaire and test data indicate that although students in the project became more aware of the modelling nature of physics, they did not score higher than their peers in 'regular' classrooms on the modelling achievement test. Coping with multiple representations, it seems, is an ability relating to reasoning skills developed by high achieving physics students in general. Skilled modellers

- display an understanding of the nature of science compatible with acknowledged views (e.g. Abd-El-Khalik *et al* 2002),
- are able to regulate their own learning and apply elaborative learning strategies when processing new knowledge in physics, and
- are more aware of, and able to decipher the use of, multiple forms of representation during physics lessons.

Conclusions and recommendations

In this paper, we have argued that modelling should be given a more prominent role in physics education, based on the nature of physics and on research on the teaching and learning of physics. We have suggested the framework with multiple representations of physical phenomena as a good basis for developing meaningful learning activities for students in a physics course focused on modelling. Moreover, we have described modelling exercises used in the project and looked briefly at the assessment of students' modelling competency.

A main conclusion from the achievement test and student questionnaire is that students' understanding of the nature of science, their learning strategies and their competency to handle multiple representations appear to reinforce each other (Guttersrud 2007). In particular, we believe that students who have acquired appropriate learning strategies are more able to 'decode the language of physics' (i.e. the use of multiple representations during physics lessons) and hence bring the teacher's and textbook's use of representations to fruition in their own processing and learning of physics. We therefore suggest that the empirical-mathematical modelling approach and the focus on representations is worth elaborating further, but that it needs to be accompanied by explicit but integrated attention to the nature of science and students' learning strategies.

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