Mental, Physical, and Mathematical Models in the Teaching and Learning of Physics

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Received 14 October 1999; revised 7 March 2001; accepted 23 May 2001

ABSTRACT: In this paper, we initially discuss the relationships among physical, mathematical, and mental models in the process of constructing and understanding physical theories. We adopt the assumption that comprehension in a particular field of physics is attained when it is possible to predict a physical phenomenon from its physical models without having to previously refer to the mathematical formalism. The physical models constitute the semantic structure of a physical theory and determine the way the classes of phenomena linked to them should be "perceived." Within this framework, the first step in order to understand a phenomenon or a process in physics is to construct mental models that will allow the individual to understand the statements that compose the semantic structure of the theory, being necessary, at the same time, to modify the way of perceiving the phenomena by constructing mental models that will permit him to evaluate as true or false the descriptions the theory makes of them. When this double process is attained concerning a particular phenomenon, in such a way that the "results" of the constructed mental models (predictions and explanations) match those scientifically accepted, one can say that the individual has constructed an adequate mental model of the physical model of the theory. Then, in the light of this discussion, we attempt to interpret the research findings we have obtained so far with college students, regarding mental models and physics education under the framework of Johnson-Laird's mental model theory. The difficulties faced by the students to achieve the understanding of physical theories did not seem to be all of the same level: some are linked to the constraints imposed to the construction of mental models by students' previous knowledge and others, linked to the ways individuals perceive the world, seem to be much more problematic. We argue that teaching should focus

A preliminary version of this paper has been presented in the VI Brazilian Meeting on Research in Physics Education, Florianópolis, October 26–30, 1998.
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Contract grant sponsor: CNPq. (Conseltio Nacional de Desenvoluimento Cienfitico e Tecnológico)

DOI 10.1002/sce.10013

on them, at least at introductory level, considering the explicit teaching of the modelling process—or at least its systematic practice—as a tool that might be appropriate to facilitate this process. *Sci Ed* **86:**106–121, 2001.

INTRODUCTION

Contributions of cognitive psychology to the understanding of learning and instruction come essentially from one of the most important research topics in cognitive psychology, namely, the study of the representational nature of human knowledge. Current research studies in this area (e.g., Vosniadou, 1996) are providing theoretical constructs much more powerful than those of former psychologies to describe mental representations and processes underlying expert performance in a certain domain of knowledge. If these processes and representations can be understood, the questions that follow might have clear educational implications: for example, are they innate or acquired (constructed)? If acquired, how were they acquired? Is it possible to design instructional methodologies that would facilitate their acquisition (construction)?

These advances in cognitive psychology together with an increasing dissatisfaction of science education researchers with the lack of significant findings (Duit, 1993) of studies concerning misconceptions and conceptual change—at least in the versions of Posner et al. (1982) and Nussbaum (1989)—have generated a great deal of interest among these researchers regarding such theoretical constructs that are being used to describe how people construct their knowledge about the world, that is, how they mentally represent it. Particularly, the idea of *mental modelling* is getting more and more attention from these researchers, both internationally—see, for instance, the review carried out by Krapas et al. (1997) on the increasing number of papers on mental models published in the most influential science education journals—and locally (e.g., Borges & Gilbert, 1998; Franco et al., 1997; Lagreca & Moreira, 1999; Moreira, 1997).

The same happened in our case: The Physics Education Research Group of the Institute of Physics of the Federal University of Rio Grande do Sul (UFRGS), Brazil, is carrying out research studies on mental models and physics education since 1994, under the framework of Johnson-Laird's theory (1983). Given this, the objective of this paper is to present an interpretative analysis of the results that have been obtained so far, concerning mental models, as well as to discuss the relationships among physical models, mathematical models, and mental models in the process of understanding and constructing physical theories. We believe this discussion is most relevant for research on physics education. Thus, we will first define what we are going to consider as these different kinds of models.

PHYSICAL AND MATHEMATICAL MODELS

A physical theory is a representational system in which two sets of signs coexist: the mathematical signs and the linguistic ones. The linguistic signs are organized into statements regarding physical phenomena, which the theory intends to describe, and they acquire their meanings in the context of such theory. However, the semantic content of a physical theory is not referred to systems, objects, or events perceived through direct observation: the relationship between theory and reality is always mediated by some physical model. *When the statements of the theory are concerned with a simplified and idealized physical system or phenomenon, the resulting description is a physical model*. Thus, in the physical model of a gas, for example, the gas is supposedly formed by a set of small balls that interact through perfectly elastic collisions. That is, the gas is not real anymore, it must be "ideal," so that the statements of classical mechanics can be applied to it.

The physical models fully develop the potentiality of the theory. They derive from images and metaphors which constrain the phenomena: if a scientific theory constitutes a particular "world view" determining the type of explanations and questions that can be formulated, the physical models determine the way the classes of phenomena linked to them should be "perceived." They determine, for instance, the simplifications, the linkages, and the necessary constraints (one may think about the applications of the point particle model of classical mechanics to any system in which a central force is exerted independently of its size), or the internal structures, even if they are not directly observed (e.g., the structures imposed by the different models of atom). They, therefore, constitute powerful heuristic "pictures," which in themselves sum up the essential aspects of the theory, so that it is possible to "visualize" with more ease through them the explanatory principles of the theory (Jammer, 1974, p. 11).

Nevertheless, the relationship that is established between reality and the physical model is complex, so that when one talks about images or "visualizations," in the context of physical models, these should be understood in their broad sense, and not as a pictorial relationship in which each element of the model corresponds to an element in reality. As said by Dirac, although the main purpose of science is not to provide images, and if they exist, or not, it is an issue of secondary relevance, one can always extend the meaning of the word *image* in such a way as to include any possible way of looking at the fundamental laws that will make evident their self-consistency (Jammer, 1974, p. 13).

On the other hand, mathematical signs shape the formalism of the theory; they are its set of statements without their semantic content. This syntactic structure constitutes what is usually called the mathematical model of the theory. Since mathematical models derive from some mathematical theory, sometimes the term mathematical model is extended to the mathematical theory from which it derives (Lombardi, 1997). *The mathematical model constitutes a deductively articulated axiomatic system, which can express the statements of the theory in terms of equations*. The values of the variables obtained through the use of these equations can be identified with the magnitude properties of the system under study only *after* its semantic interpretation through the physical model. Therefore, even if the physical model carries within it a mathematical model, this one is not in itself a description of the phenomena. This occurs because this axiomatic system lacks a frame of reference, being semantically blind (Lombardi, 1997).

Although some physicists might say that they see "the physics of a problem when it is expressed in its equations" (Greca & Moreira, 1996), it is common to accept that comprehension in a particular field of physics, is attained when it is possible to predict a physical phenomenon from its physical models, without having to previously refer to the mathematical formalism (Schenzle, 1996). (Of course, this is not true for more advanced fields of physics in which the interrelationship between physical and mathematical models is much more complex and in this case it is possible to say that one might really see the physics in the mathematics, that is, in which the application the mathematical formalism can generate new developments in the physical models.) On the other hand, Johnson-Laird (1983) believes the core of understanding lies in the existence of working models in the mind of the individual that understands, so that *the understanding of a scientific theory would require the constructions of mental models of its physical models in the mind of the one who wants to understand it*. From now on, in this paper, comprehension will be considered only in this sense.

MENTAL MODELS

A mental model is an internal representation which acts out as a structural analogue of situations or processes. Its role is to account for the individuals' reasoning both when they try to understand discourse and when they try to explain and predict the physical world *behavior*. Besides this form of internal representation, Johnson-Laird admits the existence of at least two other forms of mental representations used by individuals to “re-present” their knowledge of the world: *propositional representations* and *mental images*. Propositional representations are strings of symbols linked to each other by a particular syntax, which can be verbally expressed, and whose truth value depends on their interpretation according to a mental model. For instance, a definition or an equation, when mentally represented, needs a mental model as a referent to establish its truth value. Mental images are considered as visualizations of the model from a given perspective, that is, they are specifications of the model for particular cases. These three types of representations would have differentiated roles and structures (Johnson-Laird, 1983).

Mental models would owe their origin to the evolution of the ability of perception in organisms provided with a nervous system, and it is for this reason it is said that perception is the primary source of mental models (Johnson-Laird, 1983). Thus, human beings do not grasp the world directly but through an internal representation of it, since perception implies the construction of mental models. That is, the perception of any situation is conditioned by the mental models we are able to construct. Experiences that have already been internalized and social interactions with others constitute the other sources for models. Mental models generated by these other sources can also influence perception. Therefore, *constraints to mental model construction derive from the perceived or conceived world structure, from personal ontological beliefs, and from the need of maintaining the cognitive system free of contradictions—constraints that are products of the individual’s prior knowledge* (Sorzio, 1995). It is important to notice that mental models are dynamic and idiosyncratic structures, which have been generated to solve specific problems (Vosniadou & Brewer, 1994). However, although these models may have multiple forms for the same individual faced with the same situation, it is possible that some of them, or some of their parts, which might have been specially useful in past situations, may be stored in long term memory as differentiated structures that can be used as explanatory blocks whenever necessary.

According to Johnson-Laird, the ability to influence, control, initiate, or predict a physical phenomenon, which is at the basis of its understanding, derives from the construction of working models of this phenomenon. *When facing a situation, both the elements that have been chosen to interpret it and the perceived or imagined relations among them determine an internal representation, which is a structural analogue of the perceived reality, so that it functions as a substitute of this reality*. From the manipulation of these substitutes, properties and nonexplicit system relations appear, which can be “read” in a direct manner, facilitating the production of inferences and predictions. It is relevant to emphasize, again, that according to Johnson-Laird (1990, p. 487), in situations where there is no teaching, people are reasonably adept at constructing causal models of their own, following three principles: first, in a deterministic domain all the events have causes; second, causes precede their effects; and third, an action upon an object is the likely cause of any change that occurs in it.

On the other hand, the reason why people seem to construct mental models, when they understand what they hear or read, might be based on the fact that the explicit content of discourse usually is just a scheme of facts. It is the role of the reader or the listener to establish the relationships and the details that are missing in the set of propositional representations that constitute this discourse. The basic principle of discourse interpretation is that people construct mental models based on what they already know about the meaning of the words and about what they know that they do not know, that is, to attach meaning to new information requires the construction of mental models as well. The theory of mental models for discourse derives from the following assumptions (Johnson-Laird, 1990, p. 475): (a) a mental model, which has been constructed from propositional representations, represents the situation described by discourse; (b) the initial linguistic representation together with the mechanisms used to build and review the models of discourse capture its meaning; (c) a discourse is judged as having truth value if at least one of its mental models can be subsumed under a model of real or imaginary world; and (d) if it is not possible to construct mental models without ambiguities in relation to the situation described in discourse, people will tend to remember this situation in its propositional format. In this case, forgetting is much more faster than when it is possible to construct a mental model (Mani & Johnson-Laird, 1982). These assumptions clearly suggest the relevant role of language in building mental models and, consequently, in thinking and reasoning.

MENTAL MODELS AND THE UNDERSTANDING OF PHYSICAL MODELS

If discourse comprehension requires the construction of mental models, the comprehension of statements of a scientific theory, and by extension, the comprehension of their physical models, which are a particularization of these statements, will also require the construction of mental models. These mental models, in turn, will determine the perception of phenomena and, thus, they will condition the types of mental models that should be generated to predict or to explain these same phenomena.

This means that to understand a phenomenon or a process in physics, according to the assumption previously presented, the first step is to construct mental models that will allow the individual to understand the statements that compose the semantic structure of the theory. At the same time, it is necessary to modify the way of perceiving the phenomena by constructing mental models that will permit us to evaluate as true or false the descriptions the theory makes of them. *When this double process is attained concerning a particular phenomenon, in such a way that its "results" (predictions and explanations) match those scientifically accepted, one can say that the individual has constructed an adequate mental model of the physical model of the theory*.

This seems to be basically a process of linguistic interpretation. As Feyerabend (1988) observes, some theories—especially the physical ones—have some characteristics, which pertain to natural language. So, if someone wants to learn them, he/she must learn new perceptual and conceptual relationships, bringing into light hidden conceptions in the meanings of their statements.

The generation of new physical models or the application of the existing ones to other systems, which is the task of the scientist, requires that, together with this "semantic" process, the use of a mathematical model to provide for the translation of the phenomena into the mathematical language, which is a fundamental stage for their complete description according to scientific canons. Thus, mental models constructed this way may serve as an "intermediate level of analysis" (Nersessian, 1992, p. 58) between the phenomenon and its mathematical formulation. Actually, in the case of scientists, mental models of already existing physical models should have embedded the mathematical elements necessary to the solution of Kuhnian "puzzles" of their particular field so that when they see the phenomena, the physical model constraints and the mathematical model procedures are simultaneously imposed to them. These mental models then become "shortcuts" that are triggered in blocks. The comprehension of a scientific theory, as understood in this paper, is outlined by the concept map shown in Figure 1.

It must be kept in mind, to complete the "forest" of models, that in many physics classes, at least at secondary and introductory college levels, what students get, and must learn, are the *conceptual models* of the theories instead of the physical ones, which are not presented to them. Conceptual models are *external representations—which are precise, complete, and consistent with the shared scientific knowledge—specially created to facilitate either the comprehension or the teaching of physical models* (Norman, 1983). That is, they are supposed to be didactical version of physical models. Unfortunately, these models—such as they appear in many textbooks that became classics in physics teaching—are mainly

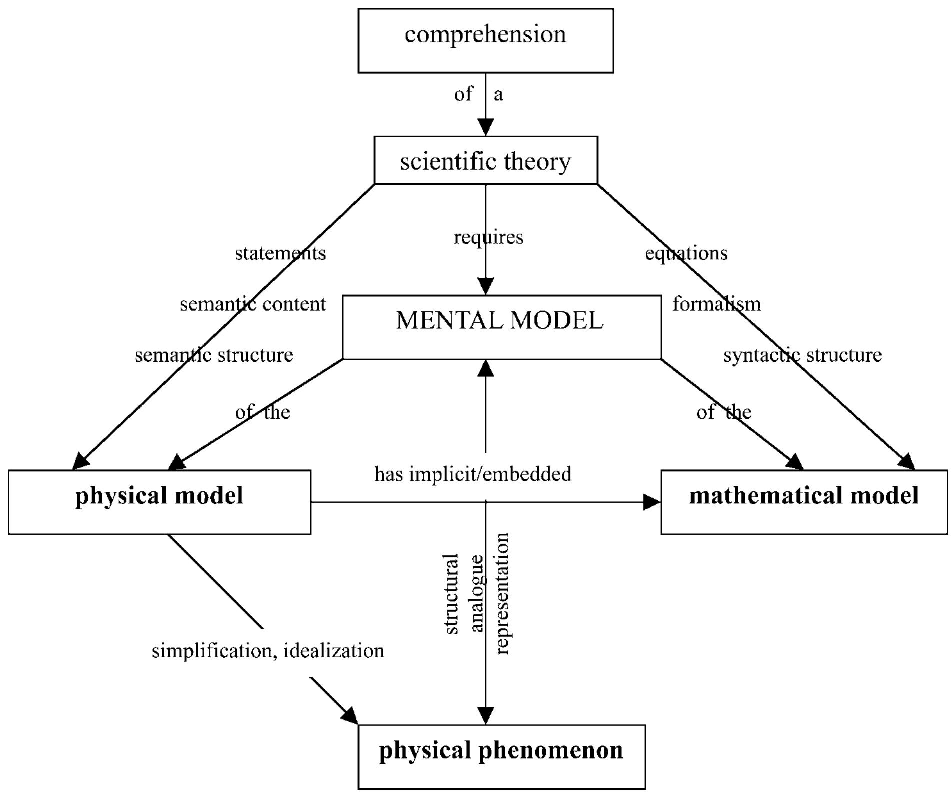

Figure 1. A concept map outlining what is meant by comprehension of a physical phenomenon in the context of this paper.

based on the embedded mathematical model, blurring out phenomenological aspects of the physical model. Conceptual models were not included in Figure 1 aiming at making it simpler. As a matter of fact, we are not going to include conceptual models in our analysis for the same reason, that is, to try to keep it as simple as possible.

It must be also remarked that, as in the case of discourse comprehension, two classes of representations are involved in the understanding of a phenomenon or a process in physics: the physical and mathematical models, which are external representations, socially constructed and shared; and the mental models, which are internal, idiosyncratic representations, whose relationship among themselves and with the world (phenomena and situations intended to be understood) are determined by representational rules and processes associated to these models (Markman, 1999). Of course, although physical and mathematical models are socially constructed and learned in social interactions their mental counterparts—the mental models of the physical and mathematical models—are specific for each individual, both in the mental organization of their content and in the inferential strategies that they determine. A dialectic process exists between these two classes of representations (Sorzio, 1995, p. 20): the inferences that might be done, from the constructed mental models, about world situations and phenomena are affected by social representations which, in turn, are comprehended, updated, and apprehended through mental models.

STUDENTS' UNDERSTANDING

The construction of mental models that allow for the grasping of the semantic aspects of theories does not seem to be an easy task, at least for the students. Findings from research on this subject, which has been carried out since 1994 by the Physics Education Group of the Institute of Physics, of the Universidade Federal do Rio Grande do Sul (UFRGS), Brazil, seem to point out to students' difficulties in constructing appropriate mental models. These research projects (Greca & Mallmann, 1997; Greca & Moreira, 1997, 1998; Lagreca & Moreira, 1999; Moreira & Lagreca, 1998) were carried out with college students, majoring in Engineering and Mathematics, taking Physics I and II at UFRGS Physics Department. Their aim was to investigate the kinds of mental representations—propositional representations, mental images, or mental models—used by students in problem solving and when answering theoretical questions linked to concepts of classical physics, in the areas of electromagnetism and mechanics, with special emphasis on the detection of possible mental models in the latter area.[1]

Research methodology was planned according to the assumption that the structure of the mental representations people use would someway have effects on their external representations. Of course, it was also assumed that these external representations would also reflect some sort of "stable" knowledge in the students' cognitive structure. For this reason, the research data of these studies, collected in real classroom teaching and learning situation, were the records that were made through a strong interaction with the students along the whole academic term, in each of the research instances. These records were the written and oral answers, together with the students' comments to problems, conceptual questions, and semistructured interviews, as well as their drawings and concept maps. The analysis of these data was basically qualitative, although in some cases the researchers approached the records quantitatively as well in order to provide for a methodological triangulation (Firestone, 1987). Data and methodological details of our studies are published elsewhere (e.g., Greca & Moreira, 1997, 1998). Our purpose here is to focus on the most relevant findings of these experiments, which are linked to the relationship among mental, physical, and mathematical models. The discussion of these relationships is presented next. As a matter of fact, most of these findings are not new and have been reported in the literature on alternative conceptions and conceptual change. However, we believe that their interpretation in the framework of mental models might be new or, at least, updated, and relevant for science education.

(a) Existence of difficulties in the construction of mental models whose products (predictions and explanations) match those of the physical models. In mechanics as well as in electromagnetism, the majority of the students were not able to construct mental models that would allow them to explain physical situations similarly to the ones that are scientifically shared, although the students had been successful in the evaluations of the corresponding subjects. At the end of the term, when they were asked during an interview about similar situations, they either indicated they did not know what could have happened or they did not know the reason why that had happened in a particular physical situation, or their explanations were not correct. In this last case, these explanations seemed to be connected to "the mental models they already had," being those the models they had for understanding the world before, during, and after the physics classes. One could say that the physics description of the world remained indifferent to the experience of the majority of the students. For example,

> Well, in this RLC circuit, I really don't know what happens. If I had the formulae, one of them might have helped me . . . (Ariel; Greca, 1995, p. 48).

[1]The proposed mental models should be understood as the researchers' conceptualizations (Borges & Gilbert, 1998).

> The large ball would hit the small one, but the small ball would not have enough strength to move the other so much. Then the small one will come back and hit the large one here, and as the large is larger and heavier, it will not have enough force to hit this one here (Patricia said this when asked about the shock of two penduli of different mass.) (Lagreca, 1997, p. 54).

(b) Use of isolated propositional representations that are not referred to mental models, reflected upon the preferred use of definitions and of mathematical formalism (or, at least, of efficient resolution algorithms) both in conceptual explanations and in problem solving. That is, students who could not construct mental models for the physical models of the theories tended to work only with formulae and definitions that, not being referred to any mental model, became quickly forgotten. At the time of the interview, by the end of the term in which they had studied mechanics, they were not able to remember the formulae associated to concepts such as linear and angular momentum. Everyday handling of them became the focus of the "learning" activity. They used equations as arguments for their explanations, and it was hard for them to predict possible physical behaviors. Obviously, problem solving gravitated around the search for the right equation, often through a trial and error process, without any intention of interpreting the problem or its result in physical terms. The students' skills, differentiating themselves in the ability to deal with the mathematical model, varied from those who knew by rote random definitions and equations to those who were able to develop efficient resolution algorithms. Nevertheless, only in a few cases, these propositionals representations were articulated similarly as they are in the mathematical model of the theory.

The reinterpretation of elements of physical models in syntactic terms performed by many of these students constituted another meaningful characteristic: the laws were understood as resolution algorithms, that is, there was not only an arbitrary manipulation of formulae as there was not even a linkage between the laws and certain phenomena.

> . . . Either I take an example from the book and learn by heart the things there, making a drawing similar to the one in the book, or I take formulae and see what sort of things I have that I can fit into them (Carol; Lagreca, 1997, p. 60).

> I define electromagnetic field based on Maxwell equations. There is a surprising analogy between electric and magnetic fields. Both are laws that have to do with the inverse of the square . . . (Christiane; Greca,1995, p. 46).

This finding is similar to those that appear in the literature regarding differences between procedural and principled knowledge: the use of isolated propositional representations would correspond to what Edwards and Mercer (1987) called "ritual knowledge."

(c) Comprehension of the physical model—that is, when students are able to construct mental models whose predictions and explanations coincide with those of the accepted physical model—lessened the importance of the mathematical model in the description of phenomena and in problem solving. In the few cases found—none in the field of mechanics—the generated mental models permitted these students to explain and predict correctly physical situations in physical terms. Problem solving, for these students, involved a previous physical comprehension of the problems as well as a later analysis of the results. The same approach was used in the laboratory experiments, where students presented a high degree of correct predictions. The equations started to play a secondary role, in the sense that although they were associated with the concepts they represented, these concepts became the objects

whose comprehension was mostly needed. Comprehension for them meant being able to explain phenomena and/or situations using just qualitative terms.

> I know how the circuits work, how the fields are modified, but I don't know by heart laws and formulae (Sandro; Greca, 1995, p. 63).

> ... I don't know how to solve this integral, although I can describe how the resulting field in this region should be ... (Paulo; Greca, 1995, p.47).

> In an oscillating LC circuit when both are 'charged' one of them must be charging and the other one discharging. When the inductor is charging it means that the magnetic field is increasing, then the electric field is decreasing. In the case of a charging capacitor the inverse situation occurs; the magnetic field is decreasing whereas the electric one is increasing. That happens because, when charging, the inductor stores energy through the magnetic field, since $U = \frac{1}{2}Li^2$ and $L = \frac{N\Phi_B}{i}$, and causes a decrease of the electrical field, since $\oint E \ \mathrm{de} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$ ... (Fabrizio; Greca, 1995, p. 65)

The findings presented so far can be summed up in a diagram such as in Figure 2.

In the upper diagram, in which dimensions were reduced to facilitate visualization, the different axes intend to represent the relevant elements for the construction of mental models that will make possible the explanation and prediction of a physical phenomenon or the solution of a problem solving. The *z* axis represents the mental model—in the case of comprehension of a physical theory, this set identifies itself with a mental model of the physical one—, the *y* axis stands for the mathematical tools, and the *x* axis represents the phenomenon or problem.

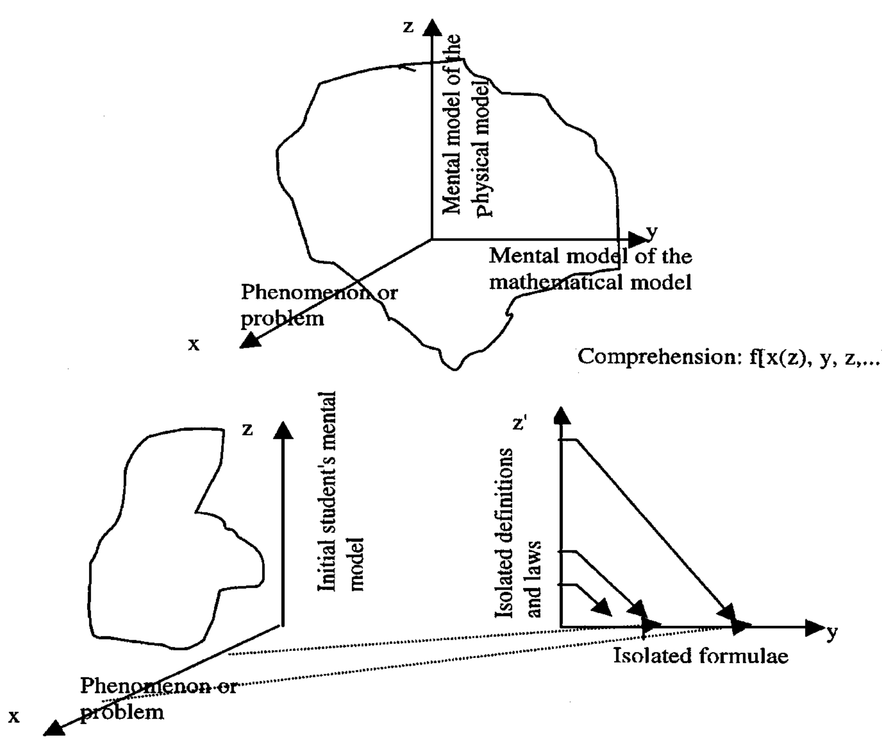

Figure 2. Diagrams summarizing the relationships among models, physical situations, and formulae.

The (x, z) plan represents the perception of the phenomenon or problem according to the mental model of the physical model, and the (x, y) plan represents the elements of the mental model of the mathematical model that allow for the solution of the problem or the analytical explanation of the phenomenon.

The comprehension of a physical phenomenon, in a scientifically accepted way (that is, construction of mental models of the physical phenomena in accordance with the physical and mathematical models of scientific theory) might be understood as a function of, at least, these elements: $f[x(z), y, z, \ldots]$.

As said before, these axes represent elements that belong to different "spaces": while axes z and y are internal or mental, the x axis is a representation of external phenomena or of the statements of a problem, and that the relationship between them are determined by the representational rules of the mental models.

In the lower diagram, however, there was an attempt to represent the instance in which there is not any comprehension of the physical theories. The z axis here represents the student's mental model (not a mental model of the physical model), and the (x, z) plan is the perception of the phenomenon (or the comprehension of the problem statement) according to this model. This will enable the student to find explanations to phenomena, even if these are not scientifically shared. The definitions and laws that form the z' axis and the mathematical formulae represented in y axis are isolated so that the students' explanation, in terms of scientific parameters (which allow him/her to pass the exams), results from rote association—represented in the diagram by the arrows—of different types of problems or explanations to certain formulae or definitions.

One can ponder that in order to enable students to construct mental models of the physical models of the theory, one should facilitate the "approximation" between axes z and z', which contain respectively the student's mental model and the physical concepts. However, the findings presented in the next section indicate that this process is not only complex but also that it might not be the most recommended.

(d) Existence of attempts, often frustrated, of recursive reformulation of the students' initial mental models to give meaning to the presented contents. According to some of our findings (Greca & Mallmann, 1997), it would seem that students recursively generate mental models based on their initial ones, in an attempt to fit into them, or to give meaning to the different contents of the subject matter. However, the concepts that serve as anchorage in the student's mental model are those common to their daily life and, just apparently, to the theory. We say just apparently common because when they are used in the theory, besides acquiring a specific meaning in its own context, the linguistic structures in which they fit are also distinct (Mortimer & Machado, 1996).

Therefore, the fact of not sharing the meanings, which is explicitly manifested by results that are not shared (correct explanations or predictions), leads the students to keep themselves at the level of the syntax of the physical model, which at least in principle does not need interpretation.

> I cannot understand how a small boy kicks the ball and there isn't any force exerted on the ball! This difficulty I have is a great one I have ... which I have learned the hard way that it doesn't ... I just can think when I put it in a formula ... (Carol; Lagreca, 1997, p. 62).

Figure 3 presents successive reformulations of mental models linked to the concept of force, according to Greca and Mallmann (1997). These reformulations derived from attempts to confer meaning to the different disciplinary contents made by the interviewed students.

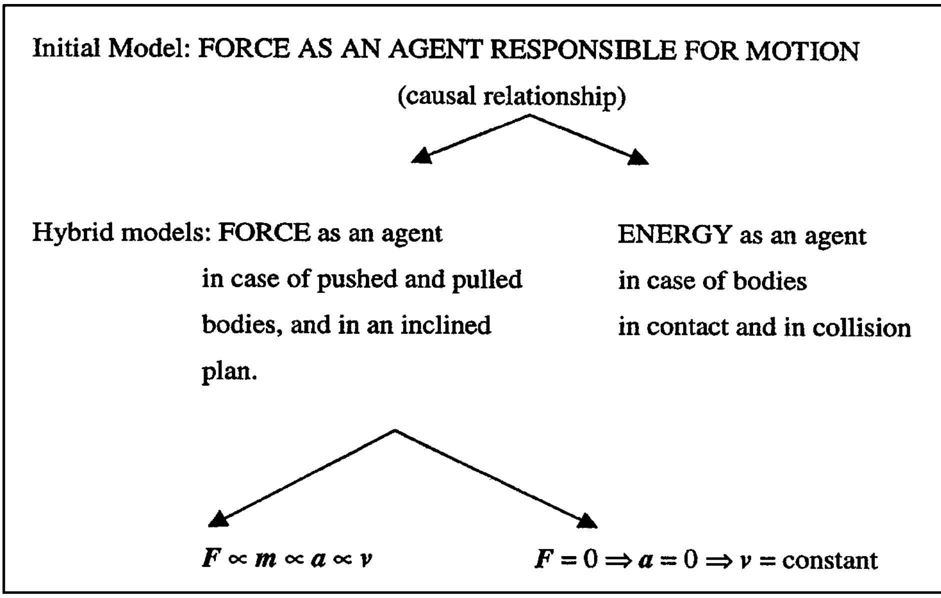

Figure 3. The initial mental model “bifurcates” itself to include the concepts of force and energy, and the original relationship between agent and motion opens up to include in it elements of Newton’s laws.

This recursive reformulation of mental models is of the same kind of the one found by Vosniadou and Brewer (1994) for the case of children’s earth mental models.

(e) The genesis of comprehensive mental models of physical models of mechanics would seem to go from the overcoming of the simple linear causality idea to the concept of force. As it appears in Figure 3, the successive models generated by the students were kept at the basis of the interpretations of everyday life experience, in the sense that although there had been changes in the initial mental model, the concept of force continued to be inserted into a matrix of simple linear causality, thus hindering its understanding as an interaction. For instance, a mental model that can be summed up by the triad $F = 0 \Rightarrow a = 0 \Rightarrow v =$ constant is always applied from left to right, and not vice versa (Greca & Mallmann, 1997). These mental models of force seem to lead the students not to understand the idea of system and, consequently, not to grasp the conservation laws.

A relevant fact here in our research in the area of electromagnetism is that the use of mental images suggested the construction of mental models for the concepts of field (electric, magnetic, or electromagnetic) (Greca & Moreira, 1998), that seemed to appear at the moment the student was able to imagine the interaction (sometimes even without using the lines of force). When he/she could imagine that interaction, he/she understood the concept of field. However, there were cases in which the students could imagine the interaction only in the case of electric field, not being able to generate it in electromagnetic field models.

The main reason of these findings would be determined basically by the constraints imposed to the construction of mental models: any new knowledge must be inserted in real or imaginary mental world models that one is able to construct, and these, in turn, are determined by his/her previous knowledge. If the information to which the subject should assign meaning does not allow for the construction of appropriate mental models to understand such knowledge, without contradicting his/her previous knowledge, then that information will be memorized as propositional representations. It means that previous knowledge acts as a filter with respect to new knowledge. However, it does not preclude conceptual evolution. In the example given before concerning models of force, the sequence of modifications provides some sort of enrichment of the initial model, even when a model whose products are compatible with those scientifically accepted is not achieved. A successful case is the one described by Nersessian (1992) regarding Maxwell who, starting with a model based on the fluid dynamics in Newtonian mechanics, constructed successive representations until he reached a model whose result—electromagnetic actions propagating with time delay—was not Newtonian.

Besides these "content filters," there seemed to be, in some cases, certain elements imposing more severe constraints to the capacity of constructing mental models of the physical models. These elements, which would be implicit in the nuclei of all possible models, would determine the mode of perception of phenomena or situations, limiting so far the possible relationships between their constituent elements, and therefore the plausible explanations. One of these elements would be the simple linear causality, which, by the way, presupposes that any explanation requires a causal agent. This may hinder, as it has been said before, the comprehension of force as interaction, thus becoming the major difficulty students face in the classes of mechanics and electromagnetism. Actually, students should start generating another "family" of models (Figure 4), whose nuclei would allow for a visualization of the interactions to enable them to understand the physical models derived from those theories. The importance of interaction schemas in scientific theories is a trait that has been emphasized by many authors from different perspectives (Chi, Slotta, & Leeuw, 1994; Inhelder & Piaget, 1975). Examples of this change of nuclei were provided by some of our Physics II students (Greca & Moreira, 1998). Those students who understood the set of phenomena associated to the concept of electromagnetic field must have come to this stage when they were studying mechanics, or during their course of electricity and magnetism—this might be the reason for a comment by one of the students who, after having constructed a model of electromagnetic field, stated that "This is the first time I can understand physics" (Greca & Moreira, 1998). Obviously, this does not mean that students who understand interaction will automatically generate mental models that are adequate to the physical ones, but it suggests that the understanding of this interaction seems to be a prior condition. It seems relevant to point out that, although the idea of interaction seems to be crucial to the understanding of mechanics and that this understanding is necessary to adequate problem solving activities, this idea is not emphasized in traditional textbooks of introductory physics courses at college level.

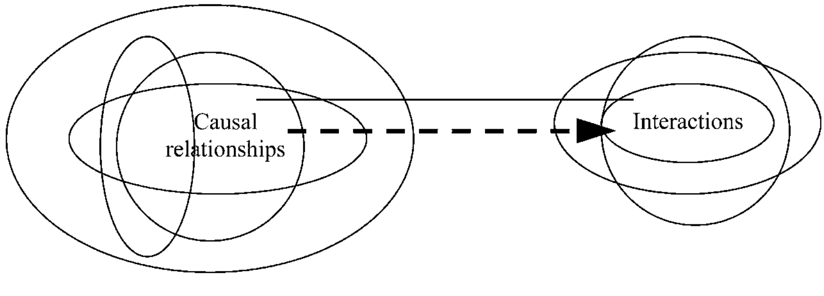

Figure 4. Two "families" of distinct mental models. The learning of mechanics would call for a passage from one of them to the other.

These nuclei[2] become the essential tokens with which mental models are constructed, shaping the form of reasoning as well as determining what is considered "intuitive." As it is much more natural for the students to look for an agent that will trigger motion in their explanations, it is natural for the physicist to think in terms of interaction and, consequently, in terms of systems. That is, once this nucleus is established, it starts determining a new intuitive form of explanation, at least in the particular field of work. It might be for this reason that conceptual models used in teaching, being a description of the physical model by its builder, do not emphasize some peculiarities of those models for considering them intuitive or evident.

A similar passage situation from one type of intuitive explanation to another, from one class of mental models to the other might occur in the transition from classical physics to quantum mechanics. Here, too, deep-rooted nuclei of a "realistic" worldview, such as locality and determinism, should be abandoned. However, when it is still possible to construct classes of mental models, whose nuclei contain the concepts of probability, nonlocality, Heisenberg's uncertainty principle, and wave–particle dualism, it would be also possible to understand the slightly intuitive quantum world (Greca & Herscovitz, 1998). Actually, in this case, the structural analogues should be understood in the broad sense proposed by Dirac.

This idea of "nucleus" of a mental model is related to the idea of basic ontological categories that determine the understanding of physical phenomena dealt by Chi, Slotta, and Leeuw (1994) in their theory of conceptual change. There they attribute the main difficulty for the conceptual change to the ontological character assigned by the students to the different concepts. In physics, for example, the students would place the physics phenomena in the substance category and not in the processes category, one of its subcategories would be the one corresponding to a causal interaction. In that way, the students would be enable to understand correctly the physics concepts. However, in our theoretical framework these "nuclei" would be associated with the innate mechanisms of construction of mental models for the physics phenomena, at least in the case of the establishment of causal relationships, more than to ontological matters.

MODELLING

Which would be the most adequate didactical strategies to facilitate the comprehension process of physical models? This continues to be the question here. Lately, a series of papers have appeared in the literature (e.g., Halloun, 1996; Nersessian, 1995; Sutton, 1996) considering the explicit teaching of the modelling process—or at least its systematic practice—as a tool that might be appropriate to facilitate this process. Nevertheless, there is no unanimity concerning what should be taught or understood as modelling, and these authors have proposed different ways of focusing it.

In principle modelling can be understood as the set of techniques used by scientists for the development of physical models, or their use in different situations. Therefore, this process also involves the perception of a system, a phenomenon, or of problems to which one can apply a particular physical model, which already exists, and its later formalization. According to what was said before, this is basically a process of construction and application of mental models of the physical ones, and for this reason it is a process with a high "semantic" content.

The learning of modelling practices by scientists seems to be made by a cognitive enculturation similar to the way a person learns his/her native language. This kind of knowledge

[2]Besides these elements corresponding to the physical model, it is possible that there are more of them connected to the mathematical models, such as the concept of function.

acquisition, which is tacit (Nersessian, 1995), is costly in terms of the time necessary to attain it and, in fact, it would seem that it is only learned by the students along their way to become physicists, and by physicists themselves. This paper, however, is not targeted at converting students into practitioners of scientific discourse, but it aims at enabling them to understand what is being talked about in the physics classes in the least time possible. That is why modelling—*understood here as a facilitating process for the construction of adequate mental models that will help understand physical models*, although maintaining some characteristics that are similar to those used by scientists—should have its own particularities.

Having this in mind and based on processes indicated as needed for the construction of mental models of the physical ones—a mental model of discourse and perception modification—it seems only reasonable that this modelling will take into account the following items:

1. As it happens in the case of a new language, the learning of the semantics of theories should precede the learning of its syntax, that is, the mathematization should be a later step and not the central one. Of course, there is a stage in which the understanding of a phenomenon requires its formalization, and it might be because of this requirement that the success of physics is due. However, as the results seem to show here, the learning of mathematical procedures in themselves—which certainly is one of the central points of physics teaching, such as it can be observed in textbooks—cannot guarantee the physical understanding of phenomena. Furthermore, the understanding of procedures would only seem fully achieved when it is linked to physical understanding.
2. However, to achieve the understanding of this semantics, that is, to allow the student to construct mental models in which the concepts of the theory acquire a scientifically shared interpretation, teaching should start with the "nuclei" of the families of models. These nuclei should have to be made explicit, discussed and "shown" to help the formation of a new "perception."[3]
3. It should also be taken into account that the students already have the basic tools to generate mental models, which are the same that they use to interpret the world: to make analogies, idealizations, and abstractions, even though to great extent they are used tacitly. The question seems to be in the learning of their explicit application to science classes.

FINAL CONSIDERATIONS

The emphasis of this paper lies on the discussion of what should be understood as the understanding of the physical models of a theory in the light of the theoretical framework of mental models, as well as on the difficulties faced by the students to achieve this understanding. These hardships did not seem to be all of the same level, since there would be some of them much more problematic, linked to the ways individuals perceive the world. Teaching should focus on them, at least at introductory level courses.

Although we have emphasized mainly the role of the individual on the process of understanding physical models, we should not put aside the social aspect implied in mental model construction, particularly in physics, which is a socially constructed and transmitted product. That is, the mental model construction of physical models is made, basically, from the interaction with others, either through the teacher or with/through textbooks.

[3] In a sense, it could be said that concept learning comes *after* the acquisition of an adequate perception.

As suggested in other parts of this text, although the structure of external representations—language and mathematical formalism structures—certainly influences the construction of mental models of physical models (for instance, the formalism resulting from Newton's equations determines the physical models of classical mechanics and, by consequence, mental models of these physical models which are distinct from those determined by the Langragian or Hamiltonian formalisms), such a question was not explicitly addressed in this paper, because we consider that the comprehension of the syntaxis comes, to some extent, after the comprehension of the semantics.

Another important aspect left aside in this analysis was related to the students' attitudes and/or beliefs concerning what the learning of scientific knowledge should be. These attitudes, in fact, impose constraints upon the constructions of mental models and a semantic emphasis is worthless if the students show attitudes connected to the belief that learning physics is to know the laws, principles, and the equations that appear on the handbooks (Hammer, 1994).

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