

Modelling simply, without algebra: beyond the spreadsheet

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Abstract

Using computers to provide dynamic modelling of physical situations is a valuable teaching tool. This is the first of two articles which look in detail at the use of two tools: this article considers the use of VnR whilst the second considers Modellus.

This article provides useful approaches using VnR to teach physics. It also considers the modelling tools themselves, their advantages and the need for teachers to be both developing these tools and searching them out. It concludes with a discussion of the place of mathematics as a way of representing patterns rather than manipulating numbers.

Introduction: choosing a tool

A large part of teaching is about making hard things as simple and accessible as possible. One way to enable a student to visualize what is happening in a complex situation is to allow them to set up a model. Using an appropriate modelling tool can make a difficult situation easier to grasp, reducing the gradient of the learning curve for a student.

To do a job well you need the right tools. For example, there are lots of things I cannot do without a screwdriver. Mathematical tools are just the same: having mastered differential equations there are lots of things I can do now that I couldn't do before, such as solving problems involving rates of change. However, we should not fall into the trap of thinking that the tool we would choose to do a job leads inevitably to the only way to do that job. Children have an everyday understanding of rates of change and, given the appropriate tools, they too can solve these problems—without differential equations.

In this article I hope to show how it is possible to build stepping stones between a child's everyday understanding and a deeper understanding of the sciences, while simultaneously exploring

and valuing patterns of reasoning. This requires thoughtfully designed activities, sensitive support and the use of the right tools at the right time.

VnR was developed as a part of the Association for Science Education's (ASE) Science Year CD series, and a copy of the modelling tool, together with supporting files, can be found on all these CDs from number 3 onwards. It was designed to support thinking in the sciences for those not yet competent with algebra, with a target audience of 11–14 year-old children. Its name is a shorthand for variables and relationships, since it encourages the user of the tool to think of the world in these terms, matching a good deal of thinking in the sciences. This article uses work from the curriculum for 11–14 year-olds and upwards: the assumption is that many students of all ages will be able to do as younger children do, and have the intuitions to build on that these young children do.

Beyond the spreadsheet

Spreadsheet modelling activities such as those reported by Lingard (2003) are widely used in the UK. One might trace them back to the introduction of the Dynamic Modelling System,

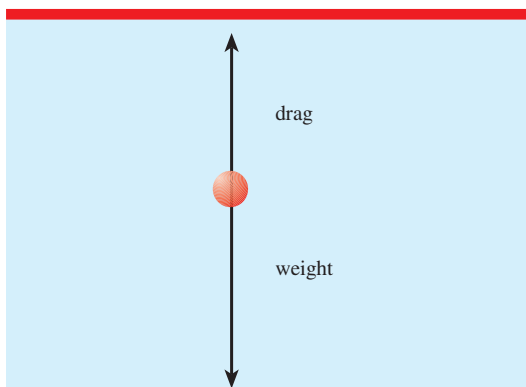


Figure 1. Simple representation of the force on a falling parachutist.

which was introduced with the revised Nuffield A-level Physics course back in 1985. These activities have tended to be used as an alternative to more mathematical solutions, supporting students with less mathematical proficiency.

This article considers teaching opportunities for students with limited algebraic skills, using VnR. The use of Modellus for those students who understand how to set up algebraic equations to describe a situation will be considered in a later article. (Between these two extremes of algebraic competence I recommend you also consider using Insight (version 4.0 and up).)

Example 1: Terminal speed

Many students are introduced to the idea of terminal speed at secondary level, based around the idea of a net force on a falling object being

Setting up a model in VnR

The representation is based on direct manipulation of what you can see. Just three clicks are needed to place the variables, four clicks link them and a final click flips the way in which the 'drag' contributes to the 'force down'. The variables are named by clicking and typing on each of them. Complete details on how to use the program are available from the online help files, accessible from within the application.

the sum of an upwards and a downwards force. A falling parachutist is a common example, as shown in figure 1. (All diagrams in this article have been redrawn from screenshots.)

I think this is a situation that can be expressed as a model in VnR. In figure 2, when the model is running, students can drag either the 'weight' or the 'drag' bars to see an immediate effect on the 'force down'—see figure 3.

Further details

Once this basic model is understood you can include the relationship between force down and speed of fall (called 'falling speed')—see figure 4. If the 'force down' is constant the 'falling speed' will increase at a steady rate, as shown in figure 5.

Increasing the drag force has the expected effect of decreasing the force down and reducing

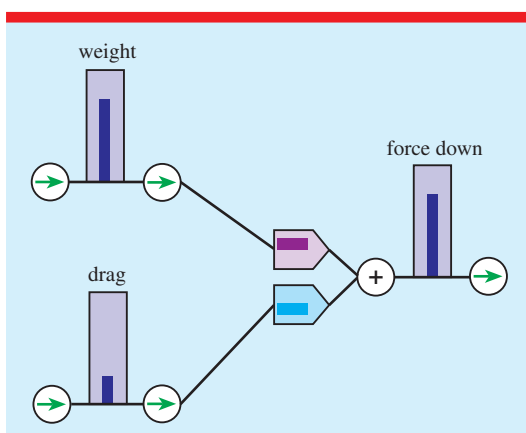


Figure 2. A model in VnR of the force on a falling parachutist.

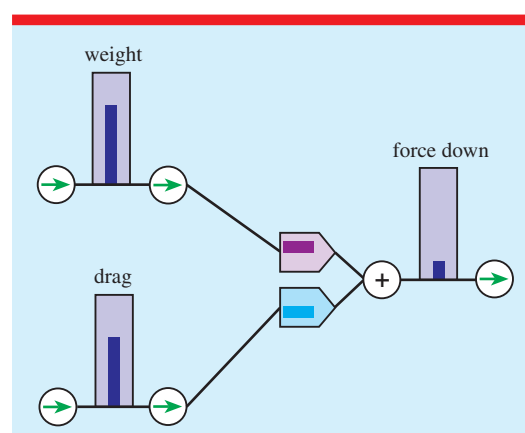


Figure 3. Using the VnR model to see the effect of increased drag.

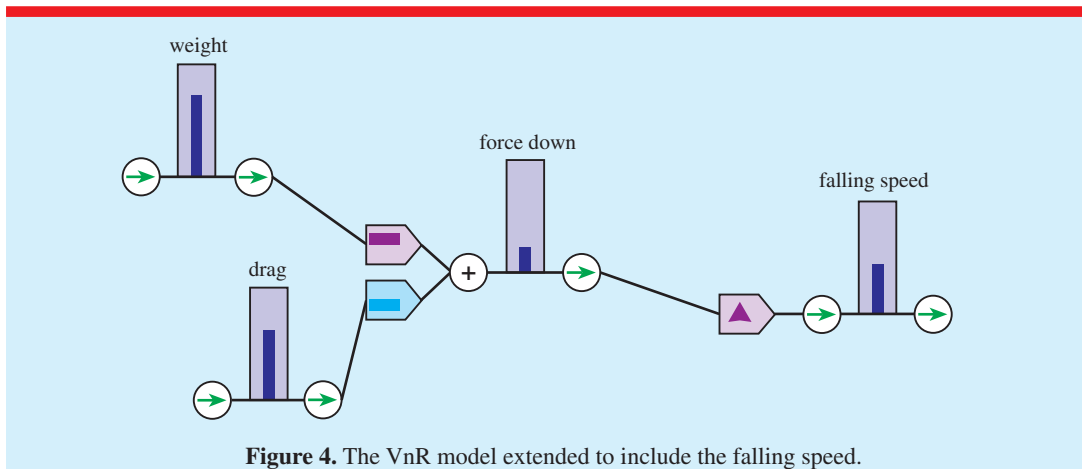


Figure 4. The VnR model extended to include the falling speed.

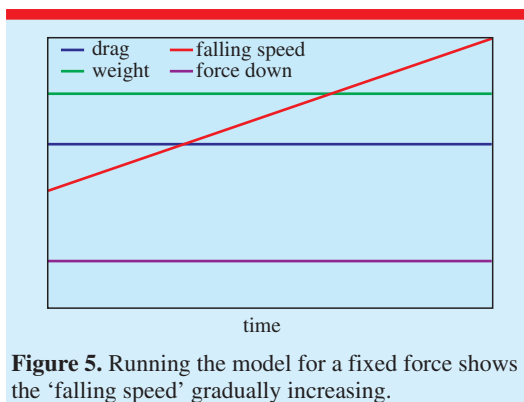


Figure 5. Running the model for a fixed force shows the 'falling speed' gradually increasing.

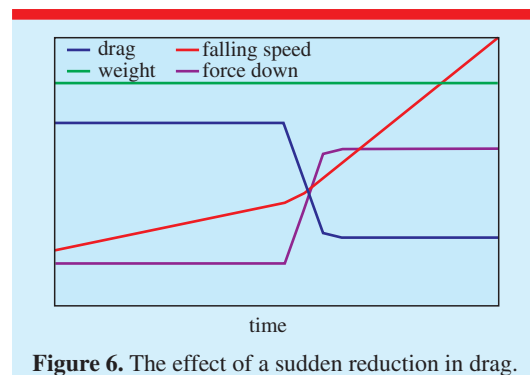


Figure 6. The effect of a sudden reduction in drag.

the rate at which the speed increases. The software provides the intermediate steps between the change in drag force and the change in speed of the parachutist: the step and their connections are transparent and intelligible. This is best reinforced by allowing students to fiddle with the input variables (the drag force and the weight).

Manipulation underpins understanding, so the objects of thought with which the situation is being analysed are open to direct manipulation. Students can be encouraged to see what happens when there is a sudden reduction in drag while the model is running (figure 6).

Now comes the nice bit. We know that speed affects drag and we want to show this in the model. This program obliges, providing a direct pictorial display of feedback, showing how the speed contributes to the drag with a line from the right back to the left-hand side (figure 7).

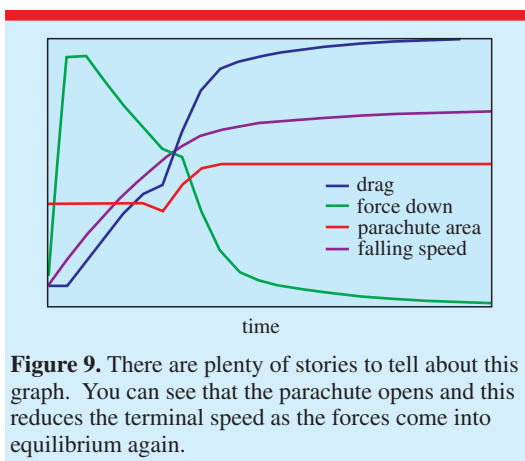
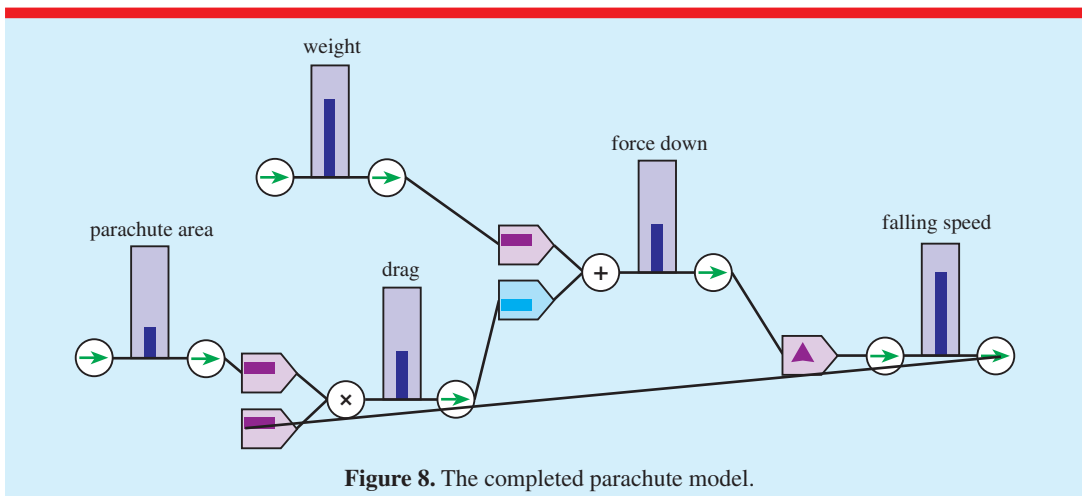
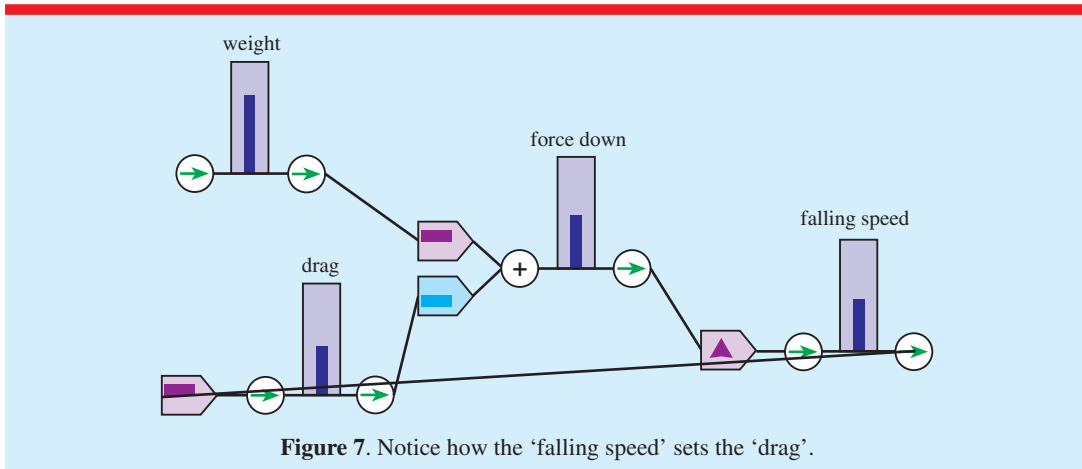
Finally, to finish the model off, I want to be able to open and close a parachute so that it

affects the drag force. This one is a little different from the other relationships and more subtle. If the parachute has no area the drag will always be zero, whatever the falling speed. If the speed is zero the drag will be zero, whatever the parachute area. 'Falling speed' and 'parachute area' must be combined by multiplying, rather than by adding, to get this result—see figure 8.

Running the model allows you to see how the inputs affect the outputs, and how the feedback leads to a steady final state. Simultaneously you can see the values of all the intermediate variables and all relationships and combinations: nothing is hidden. You can also get an overview of how the variables change with time, by pulling up a graph of up to four variables against time (figure 9).

Interlude

If you are brave and enjoy theorizing in science you may like to take a break from this article and turn to the software. Using a spreadsheet model



(such as discussed by Lingard 2003) or just being guided by your previous experiences, try working

out narratives you could share with a class as you deal with capacitor discharge and radioactive decay. See how much pleasure there is to be had from this kind of exploration! You should aim to provide appropriate guidance for children, so that they can enjoy this too.

Example 2: Capacitor discharge and exponential decay

Well, if you are still here, this is how you can get to grips with some further ideas using VnR.

Capacitor discharge and exponential decay are taught at post-16 pre-university level in the UK. Capacitor discharge may well remain at this level because of the background knowledge needed to build up the model. However, a serious understanding of exponential decay may well

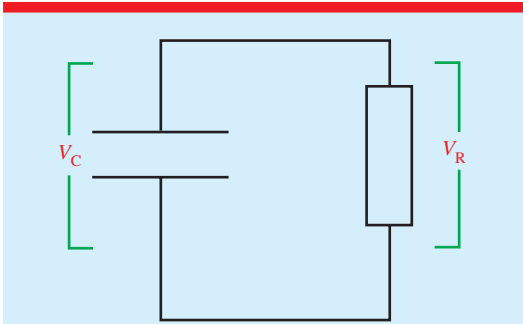


Figure 10. The circuit to consider. Note that this diagram uses the convention of putting potential differences in the vertical plane with sources on the left and sinks on the right. Consistent use of such conventions can be very helpful to students.

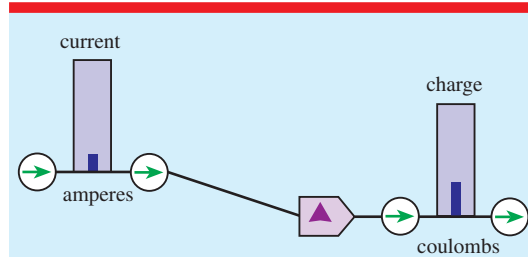


Figure 12. Current is the rate of flow of charge.

migrate to pre-16 classes if students get the right software to help them understand it.

Let's think about how we should present capacitor discharge using this semi-quantitative modelling tool: we want to be able to express the steps in the process of discharge, so that our students understand it better. The detailed connections need to be made individually by each student. Consider the circuit in figure 10.

$V_C R$ can be used to model the two simple equations that students need to have learned about current and capacitance—see figure 11. By running the model we can review these behaviours, reinforcing the fundamental relationships on which the understanding of discharge is built.

Now we can introduce another relationship—that between charge and current, shown in figure 12.

This is similar to the parachutist model: note the very simple way that a steady current produces a steady increase in the charge that has accumulated. Now this relationship needs to be added to that in figure 11. This alters the sense of the accumulation, as current reduces the charge on any one plate of the capacitor (figure 13).

There is one more link to make the model complete. The potential difference across the capacitor must equal the potential difference across the resistor, so we get figure 14.

In this completed model the feedback loop is clearly controlled by simple physics. The behaviour of each circuit element is clearly represented and the physical condition that the p.d.s are equal is at the core of the model. You can alter the value of any variable whilst the model is running and see the effect on all other variables.

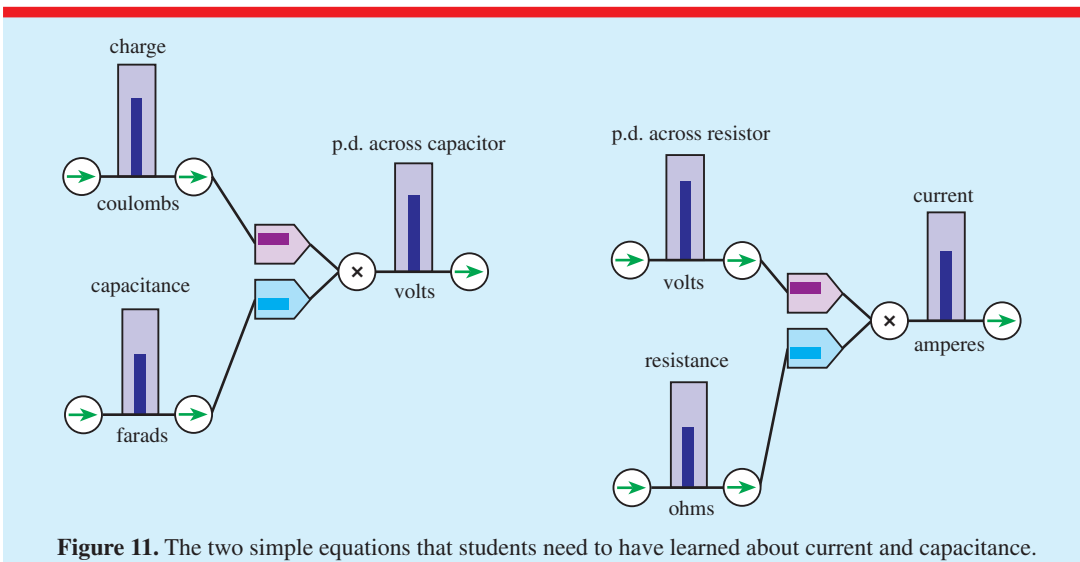


Figure 11. The two simple equations that students need to have learned about current and capacitance.

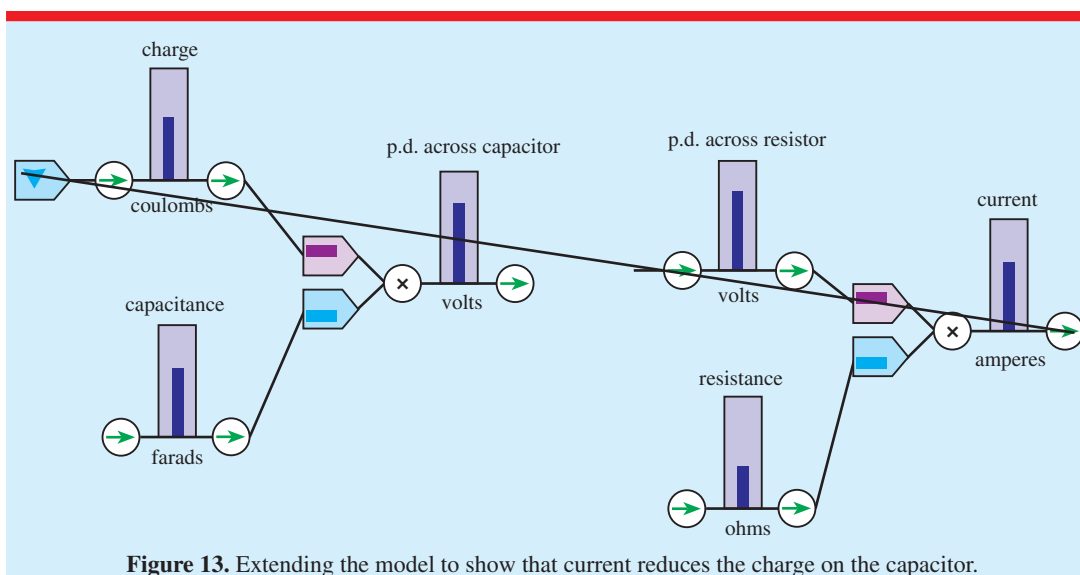


Figure 13. Extending the model to show that current reduces the charge on the capacitor.

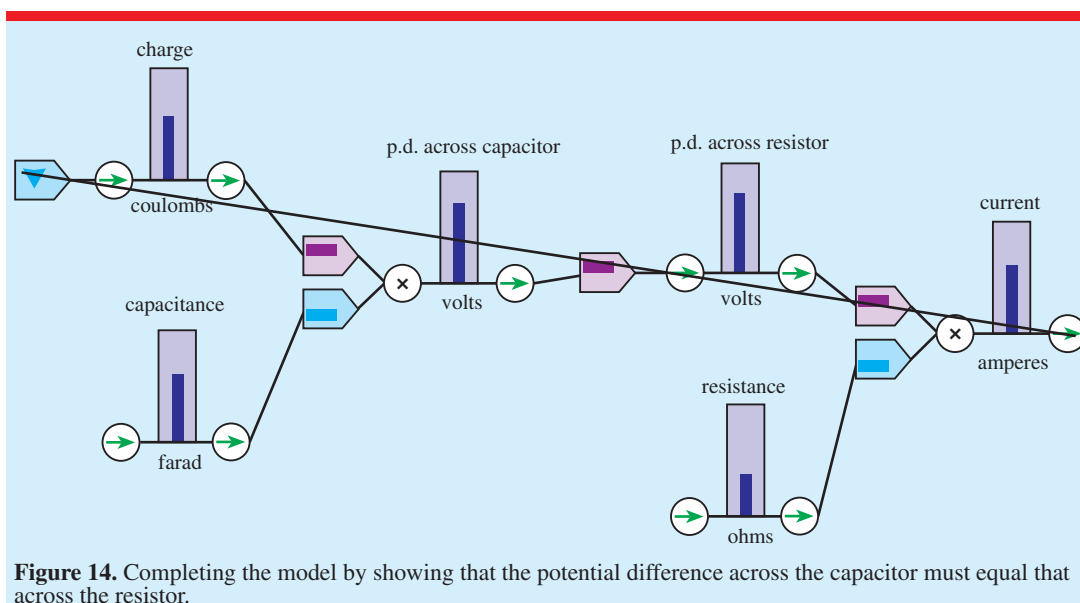


Figure 14. Completing the model by showing that the potential difference across the capacitor must equal that across the resistor.

Figure 15 shows an example of a discharge curve of a capacitor.

I am sure you will enjoy exploring the effects of changing the resistance and capacitance, even in the middle of a discharge if you like. You can have those semi-quantitative arguments about the size and direction of current/p.d. and so forth that are so important in building real physical understanding.

Radioactive decay is a much simpler case. You need less theoretical infrastructure to understand the significance of the model and how it is

built. In fact, the problem may be that there are too few parts! All we need is a variable that affects the rate of decay of that variable. Unfortunately that may mean that the simplest model is difficult to grasp conceptually—if we jump straight to the solution as illustrated in figure 16.

It may be better to build up the model step by step, deliberately putting in more steps than necessary for a solution. You might start by suggesting that the number of decays is a constant fraction of the number of nuclei, as illustrated in figure 17.

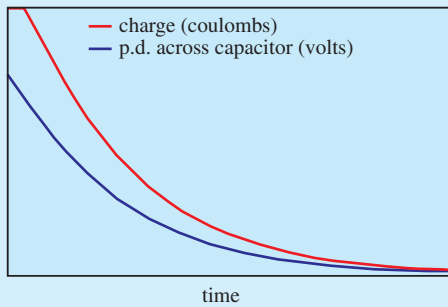


Figure 15. An example of a discharge curve of a capacitor, as in the model in figure 14. In this case you can see that I resisted the temptation to fiddle with the circuit values whilst the discharge occurred.

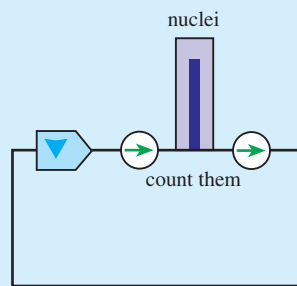


Figure 16. The simplest model of radioactive decay.



Figure 17. The number of decays is a constant fraction of the number of nuclei.

The next step is to relate these decays to the number of nuclei present in the sample. A constant number of decays reduces the number of nuclei at a constant rate. When a feedback loop is added we finally get the exponential decay that models the radioactive decay of nuclei (figures 18 and 19).

This final model could be an end point, but you may like to note that the constant fraction, introduced in figure 17, could appear elsewhere in the model, simply by setting up the model so that the number of nuclei controls the rate

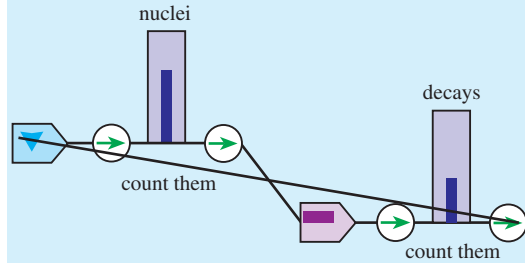


Figure 18. Adding a feedback loop.

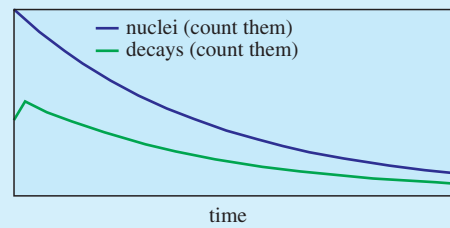


Figure 19. This graph shows the results from the model in figure 18.

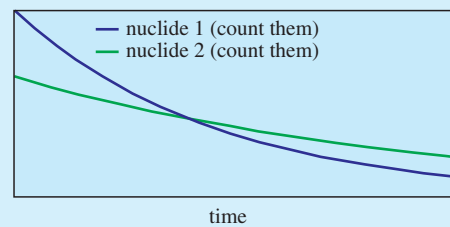


Figure 20. The behaviour of two radionuclides of different activity, represented by the simplest model possible.

of decay more weakly. Or you might want to compare the rates of decay of two separate species of radionuclide (figure 20).

The model in figures 16 and 21 shows the feedback loop so simply that it is almost brutal. You might decide that this simplification is a step too far for your students, but it could prove very useful if you want to move to writing $\Delta N/\Delta t = -\lambda N$.

A choice of tools

Why use a modelling tool such as VnR rather than a spreadsheet such as Excel? There are several advantages—not surprising if you consider what the two tools were originally designed for! These are:

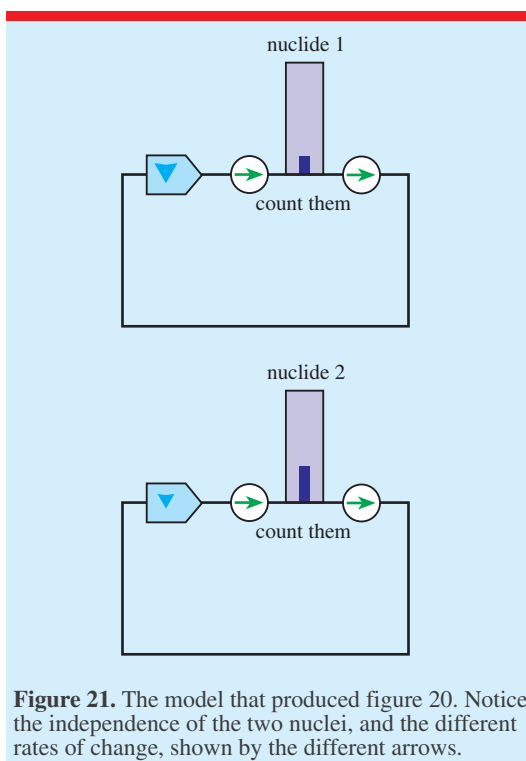


Figure 21. The model that produced figure 20. Notice the independence of the two nuclei, and the different rates of change, shown by the different arrows.

- clear variation of the intermediate variables,
- a visual picture of the feedback loop,
- students can maintain a strategic view of all the steps in the argument. (Doing this is at least as difficult as making each step.)
- simple access to the fundamental building blocks as the argument is constructed.

In contrast with a spreadsheet, students using a model like this do not have access to any actual numbers. This is deliberate—the aim is to focus on the patterns in the mathematics rather than the calculations.

You can drive a screw into a piece of wood with a hammer, but once you have learnt to use a screwdriver this becomes the tool of choice. I think our children deserve to have access to reasonable tools, and even to have some choice, learning to pick the best tool for the task. VnR does not solve all problems: we need something that clearly depends on numbers and links more formally to algebra. In a second article I intend to show how Modellus does this. VnR also restricts us to modelling with variables and their relationships. Other modelling tools might allow us access to other building blocks.

Beyond us and them

The importance of notation in mathematics cannot be overemphasized. A well chosen notation suggests the right operations and liberates the mind from pointless distractions, while an ill-chosen symbolism may be a hindrance to reasoning. From the point of view of logic and rigour, notation is irrelevant, but it has a bearing on the relationship between imagination and formalism. An efficient notation should be suggestive, meaningful, suited to our imagination as much to the subject matter.

(Omnes 1999)

When teachers talk about their students there is a tendency for them to say that some things are ‘Beyond them mathematically’. However, I contend that this depends very much on the notation method that you choose. Here are a few sums. Like me you will probably find some to be more difficult than others:

$$\begin{aligned} \text{XVI} \times \text{XVI} &= \\ 2^4 \times 2^4 &= \\ 16 \times 16 &= \\ 10\,000 \times 10\,000 &= \end{aligned}$$

Given the correct interpretation of the symbols these calculations are, of course, all the same. Yet trying to solve them within the suggested systems is not equally easy. Searching for easier ways to ‘do the same thing’ (or doing something close to the same thing, for the mental operations are clearly not identical) is a vital, and really enjoyable, part of being a teacher.

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Ian Lawrence investigates novel approaches to science teaching at the University of Birmingham. He was one of the editors of the IOP’s Advancing Physics course and in 2003 received the IOP’s Bragg medal for his innovative contributions to physics education.