Realistic Mathematics Education: history and principles

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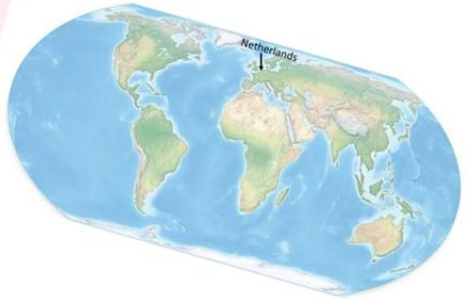
Content of today's lecture

1. History and context of Realistic Mathematics Education

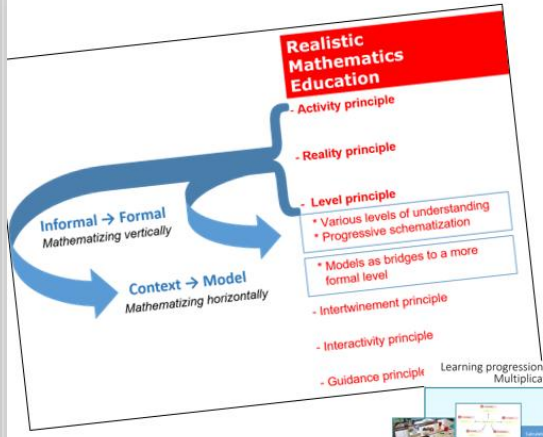
What is Realistic Mathematics Education?

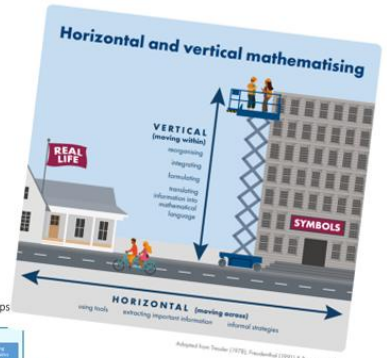
Realistic Mathematics Education (RME) is a domain-specific instruction theory on the teaching and learning of mathematics...

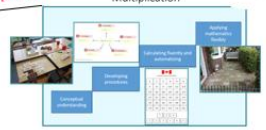
... that has been elaborated into a number of local instruction theories for different mathematical topics, student ages, and achievement levels.



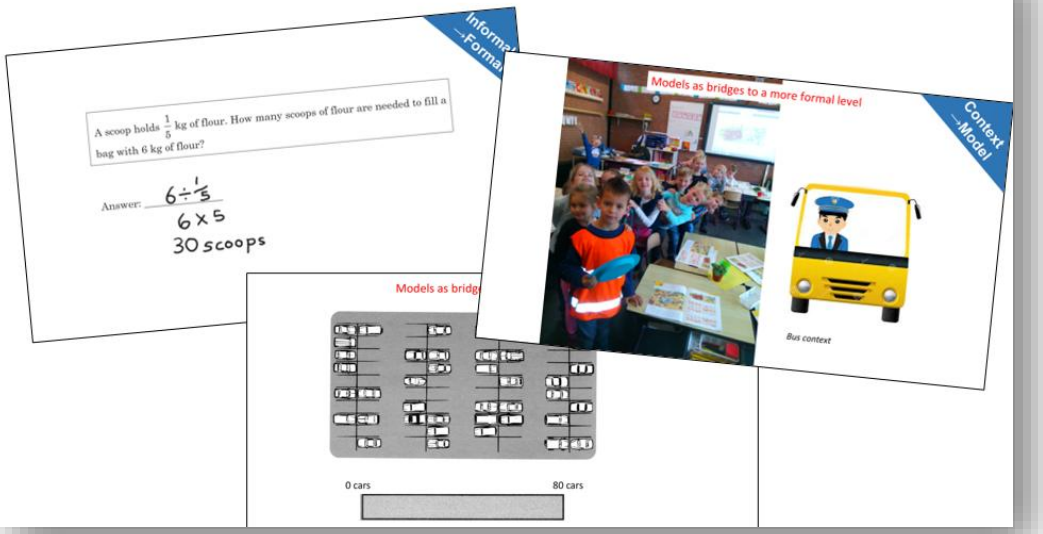
2. Principles of Realistic Mathematics Education







3. Illustrations



4. Next lecture: Big Mathematics Day

- See you in our next lecture: **October 7th**
- On the translation of these principles into the Big Mathematics Day
  - Doing mathematics in coherent activities in and outside the classroom
- With examples of such activities of different Big Mathematics Days in the Netherlands

Content of today's lecture

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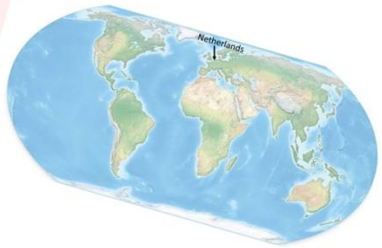
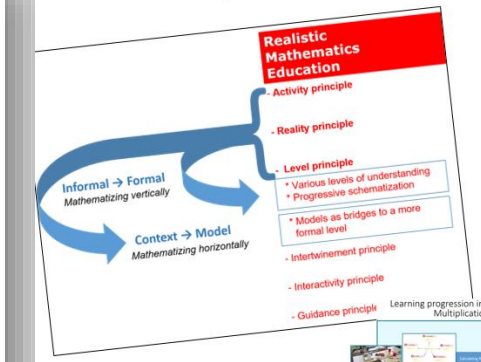


2. Principles of Realistic Mathematics Education



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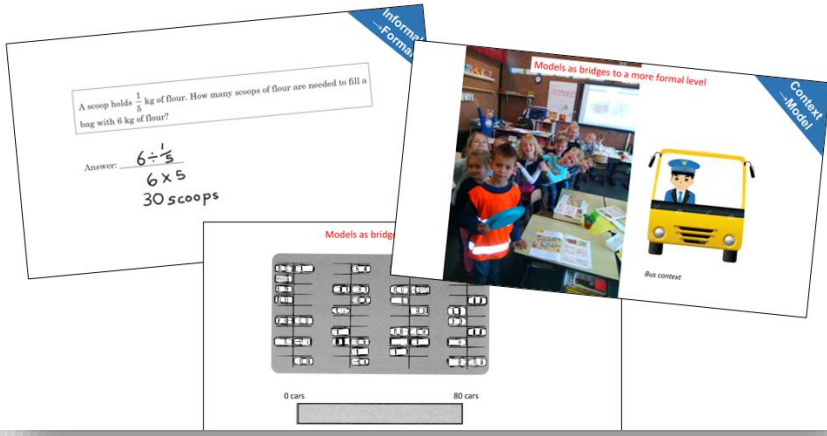

Informal → Formal

A scoop holds $\frac{1}{5}$ kg of flour. How many scoops of flour are needed to fill a bag with 6 kg of flour?

Answer: $6 \div \frac{1}{5}$

6×5

30 scoops

Context → Model

Models as bridges to a more formal level

Bus context

Context → Model

Models as bridges to a more formal level

Mathematics in Context
Grade 5

Romberg & De Lange (Eds.)
(1997-1998 ...)

*"Making sense of **percents**"*

1. History and context of Realistic Mathematics Education

SCHOOL
pabo

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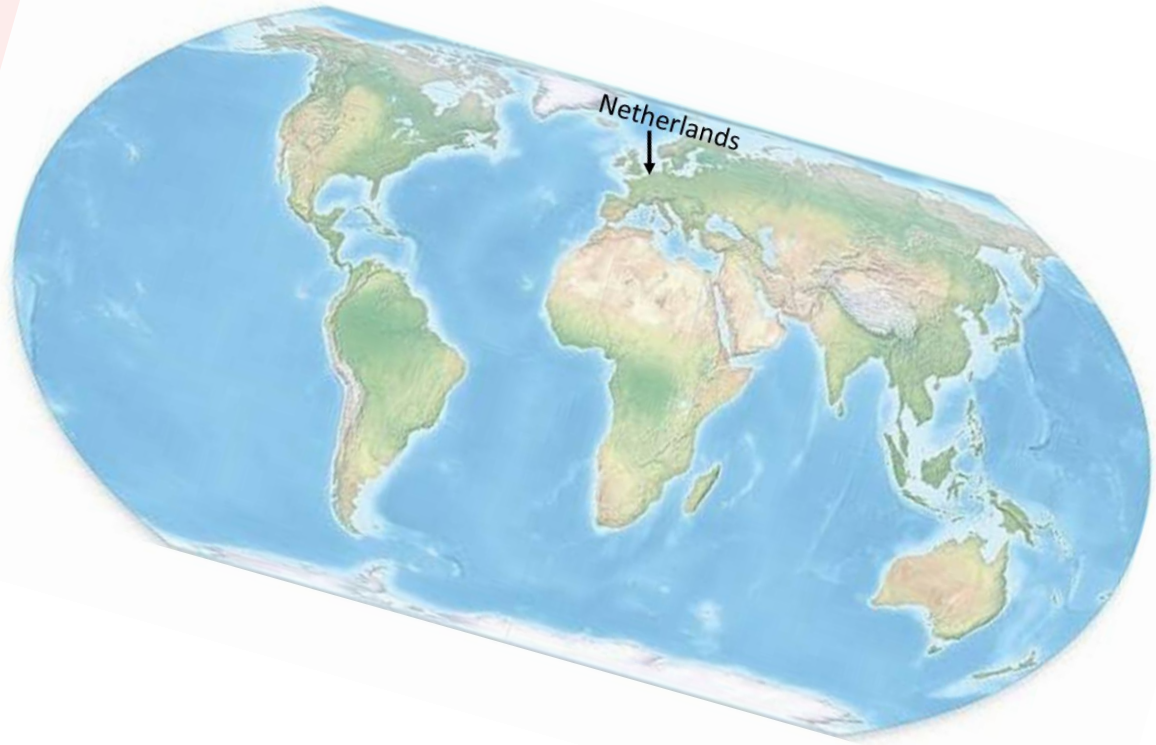

Netherlands
China

Netherlands

NORTH
SEA
Groningen
Zwolle
AMSTERDAM
Enschede
Den Haag
Utrecht
Rotterdam
Eindhoven
N

HOGESCHOOL ipabo

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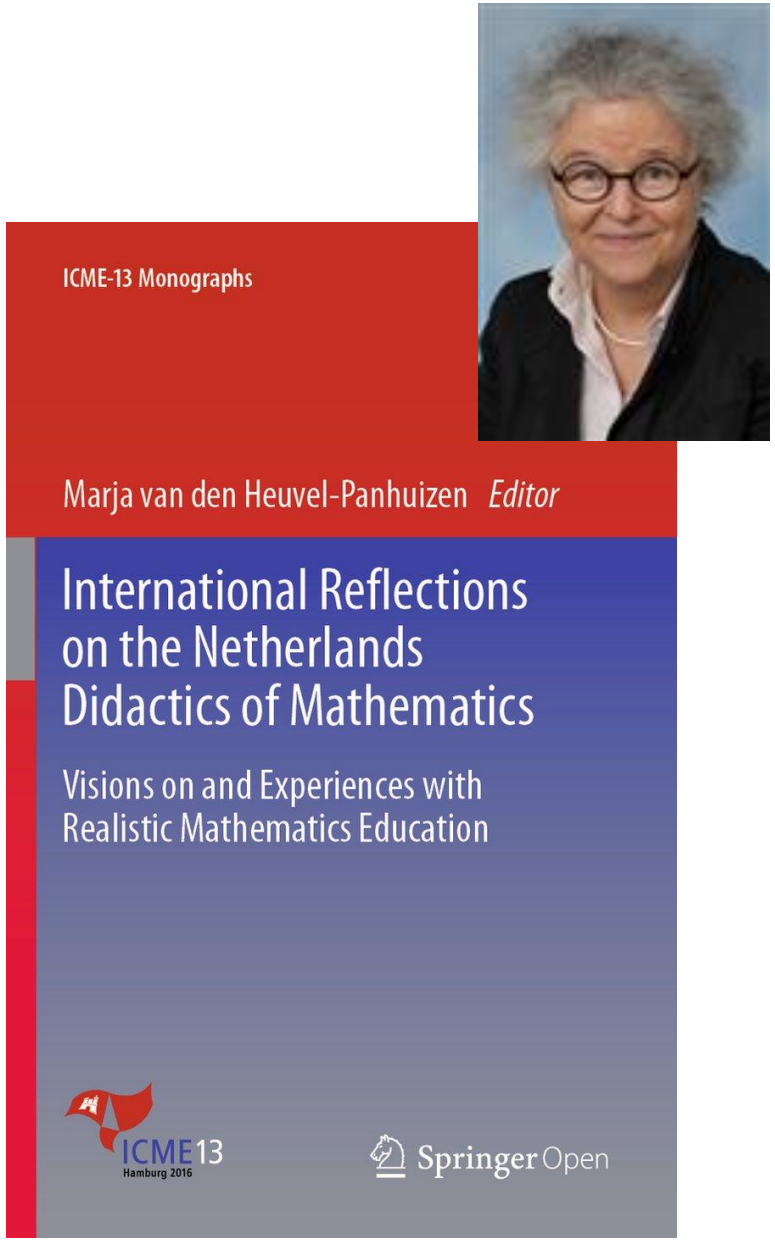


Chapter 10
Realistic Mathematics Education in the Chinese Context—Some Personal Reflections

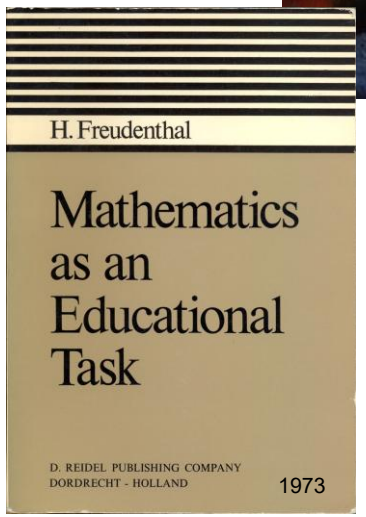
Xiaotian Sun and Wei He

Abstract In this chapter, we start with a historical review of how Professor Hans Freudenthal and Realistic Mathematics Education (RME) became known in China, and how the academic exchange between Chinese scholars in the field of mathematics education and researchers at the Freudenthal Institute initiated and continued later. Then we discuss the positive impact of RME. Specifically, we cite some living examples for how the theoretical and empirical research substances related to RME-influenced mathematics curriculum development in China. These examples include the fields of curricular policy making, textbook design and classroom teaching.

Keywords Hans Freudenthal · Freudenthal Institute · Realistic Mathematics Education · Curricular policy · Textbook design · China

Professor Hans Freudenthal

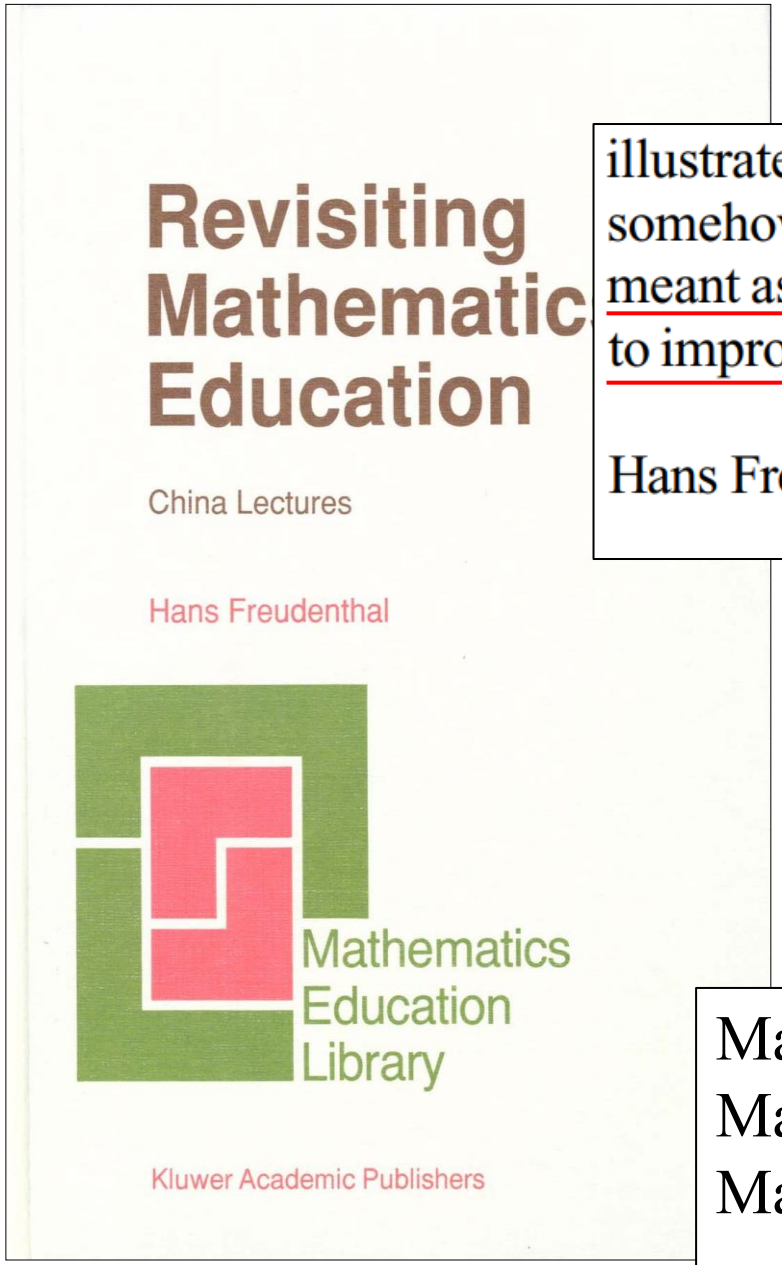
Professor Changpei Wang



1986 Meeting at the CIEAEM conference in London

Wang visited the Netherlands

1988 Freudenthal visited China



illustrate my view on mathematics education in all its aspects. The present book somehow reflects these talks, albeit in no way textually. Therefore the subtitle, meant as an homage to my listeners who, by their critical curiosity, induced me to improve on clarity and concreteness.

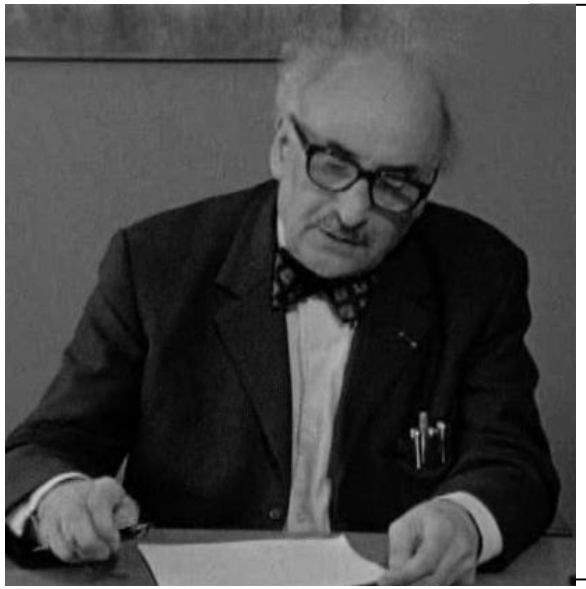
Hans Freudenthal

(p. ix)

Mathematics has arisen and arises through mathematising [...].
Mathematising is mathematising something [...]
Mathematising is mathematising reality

(p. 66-67)

Mathematics as a human activity

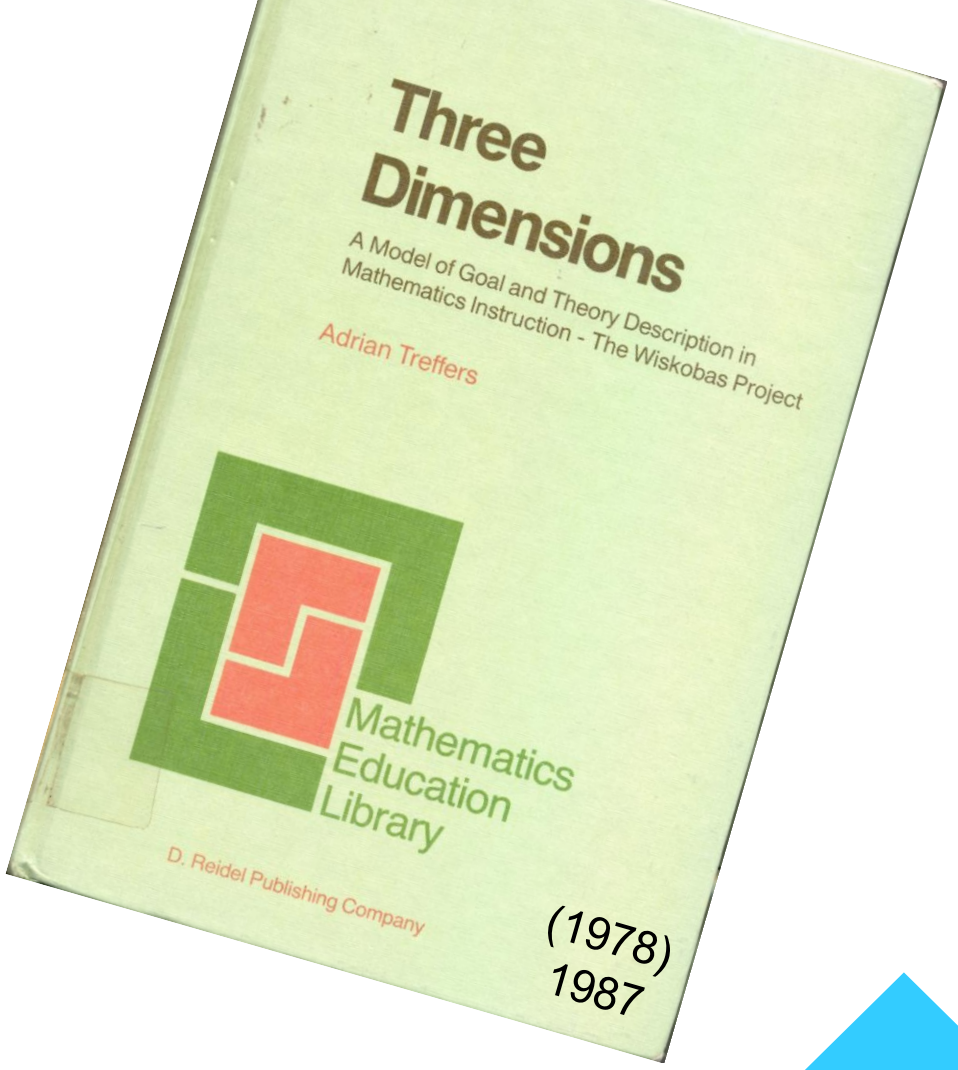


"What humans have to learn is not mathematics as a closed system, but rather as an activity,
the process of **mathematizing reality**
and if possible even that of
mathematizing mathematics."
(Freudenthal, 1968)

Beginning with abstractions and definitions to be applied later

Beginning with **rich contexts** that ask for **mathematizing**

Professor Adri Treffers



mathematizing

"real" world → **1** → **mathematics** → **2**

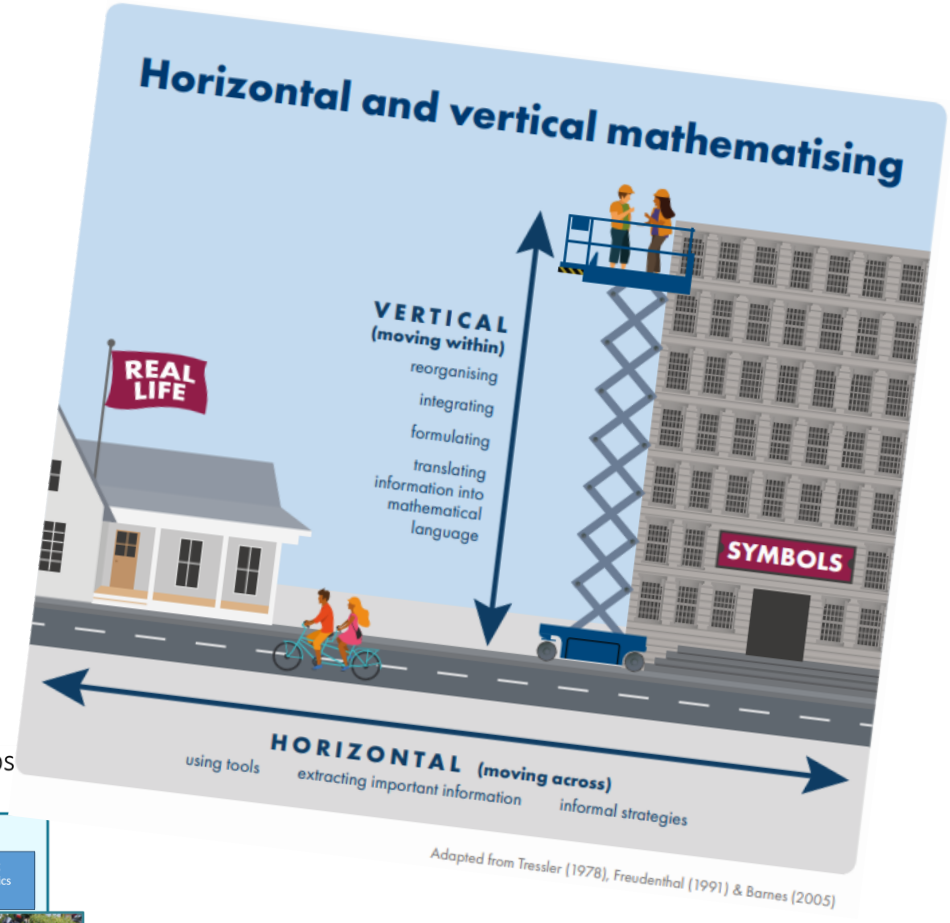
2. Principles of Realistic Mathematics Education

Realistic Mathematics Education

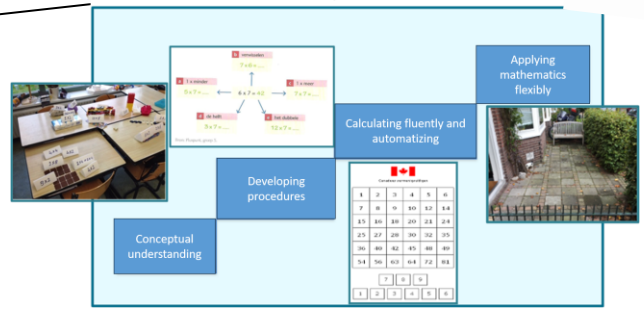
- Activity principle
- Reality principle
- Level principle
  - * Various levels of understanding
    * Progressive schematization
  - * Models as bridges to a more formal level
- Intertwinement principle
- Interactivity principle
- Guidance principle

Informal → Formal
Mathematizing vertically

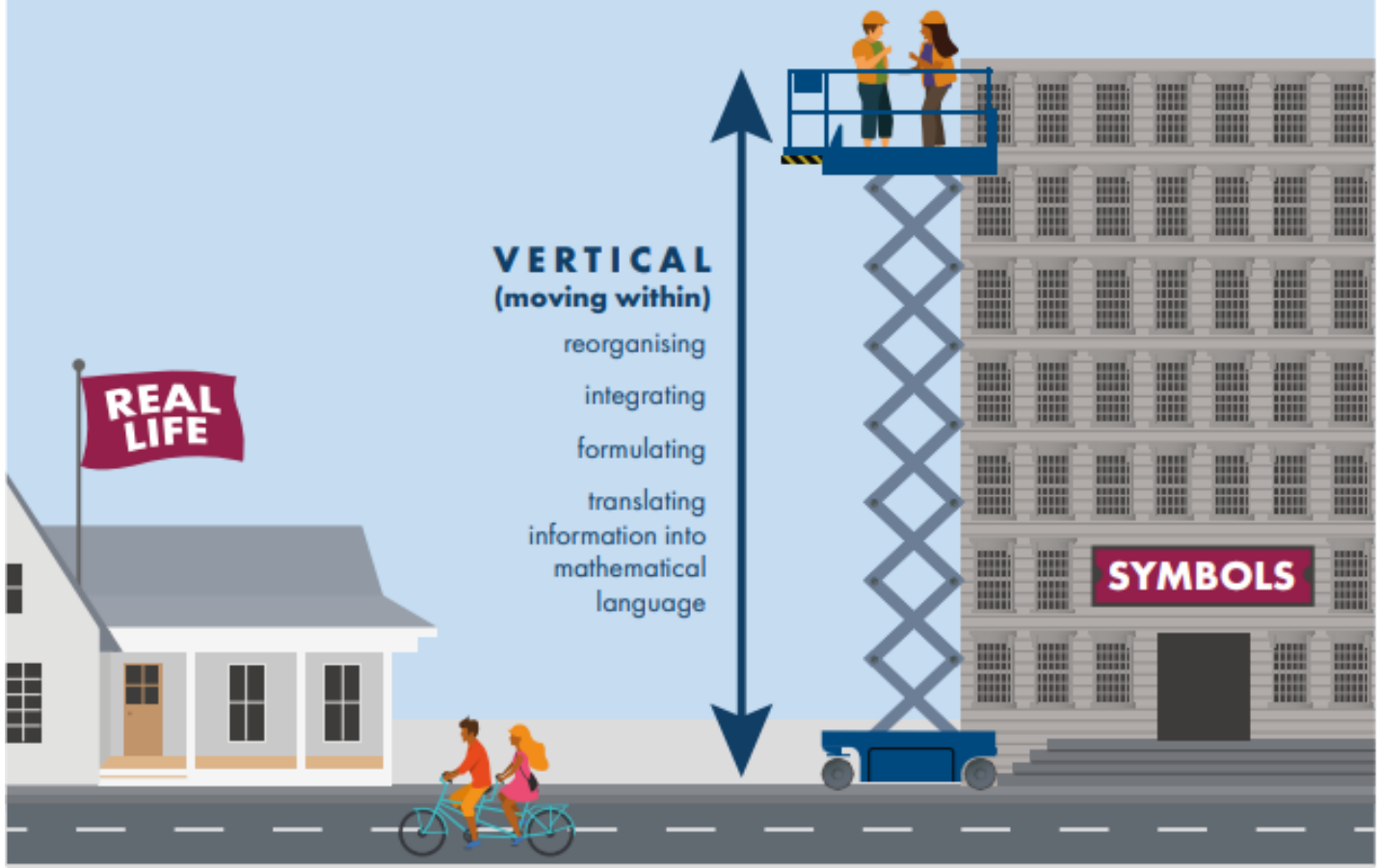
Context → Model
Mathematizing horizontally



Learning progression in broad steps
Multiplication



Horizontal and vertical mathematising



VERTICAL
(moving within)

reorganising

integrating

formulating

translating information into mathematical language

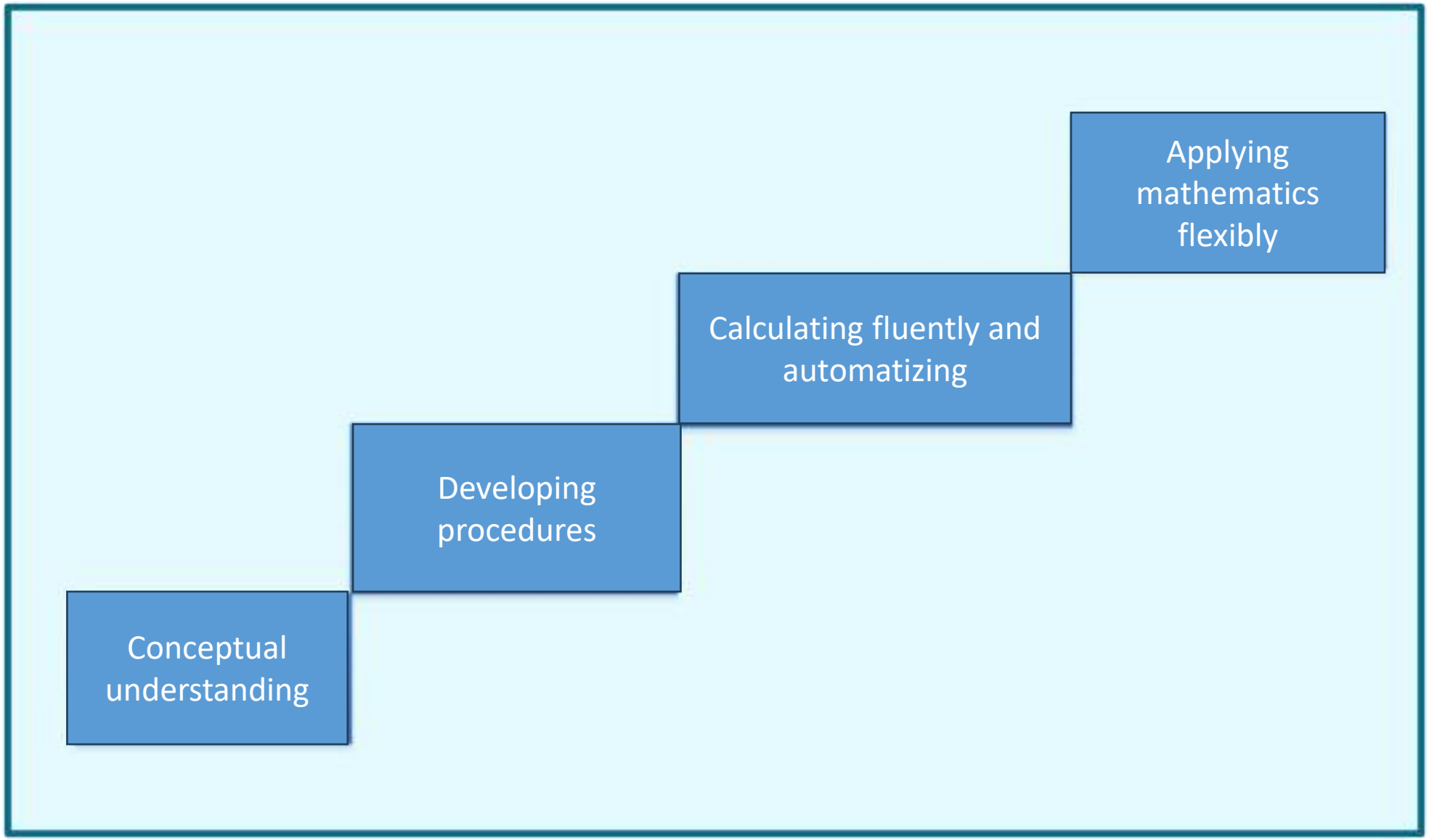
HORIZONTAL **(moving across)**

using tools

extracting important information

informal strategies

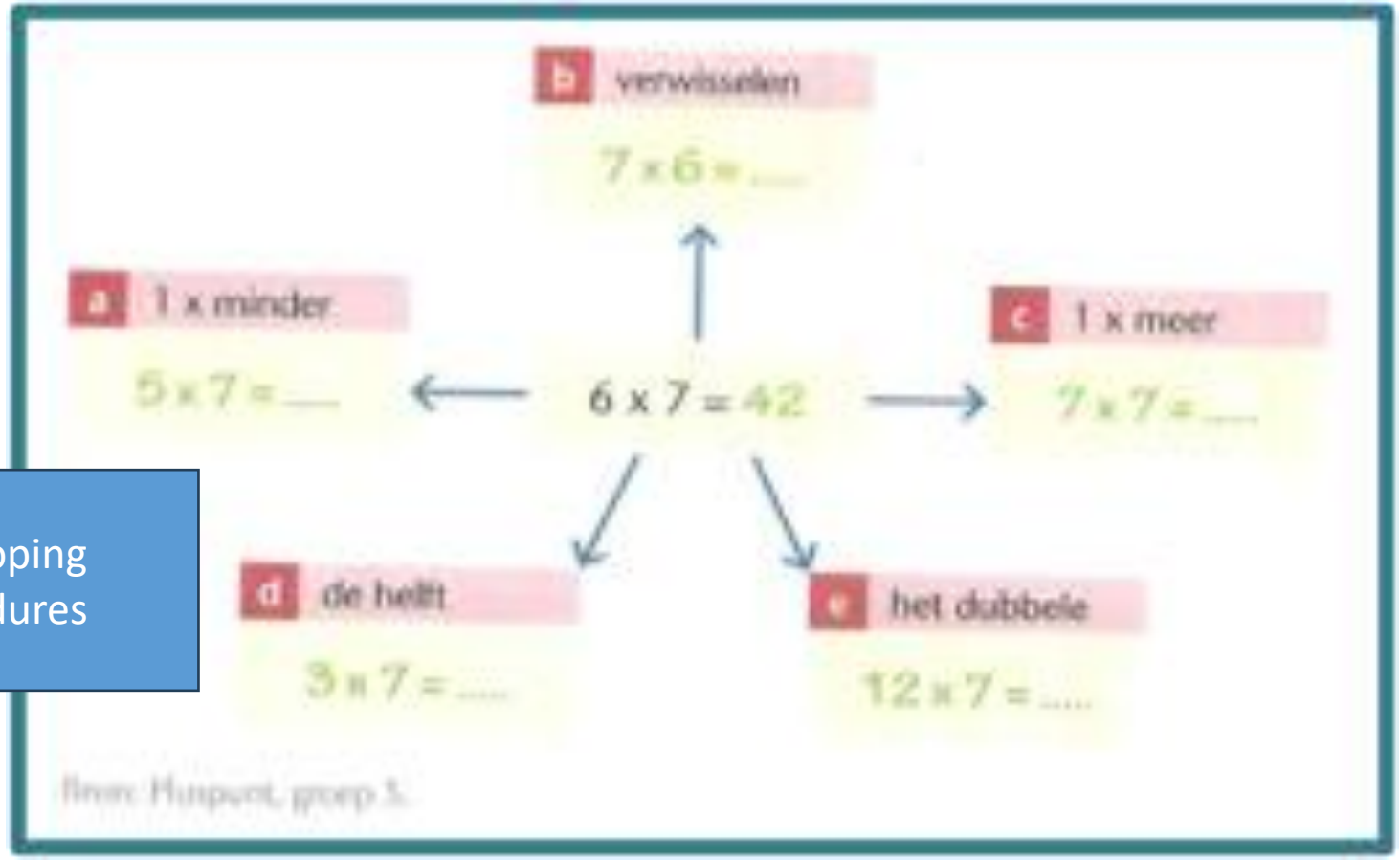
Learning progression in broad steps



Learning progression in broad steps: Multiplication

Conceptual understanding

Learning progression in broad steps: Multiplication



Developing procedures

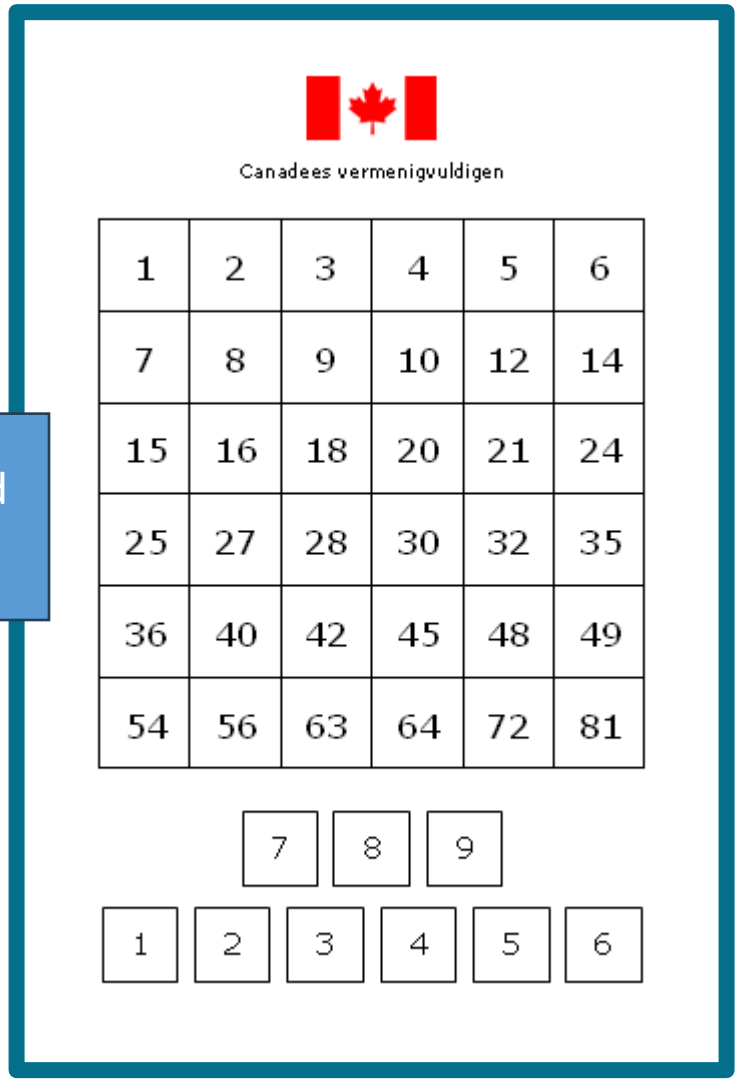
Conceptual understanding

Learning progression in broad steps: Multiplication

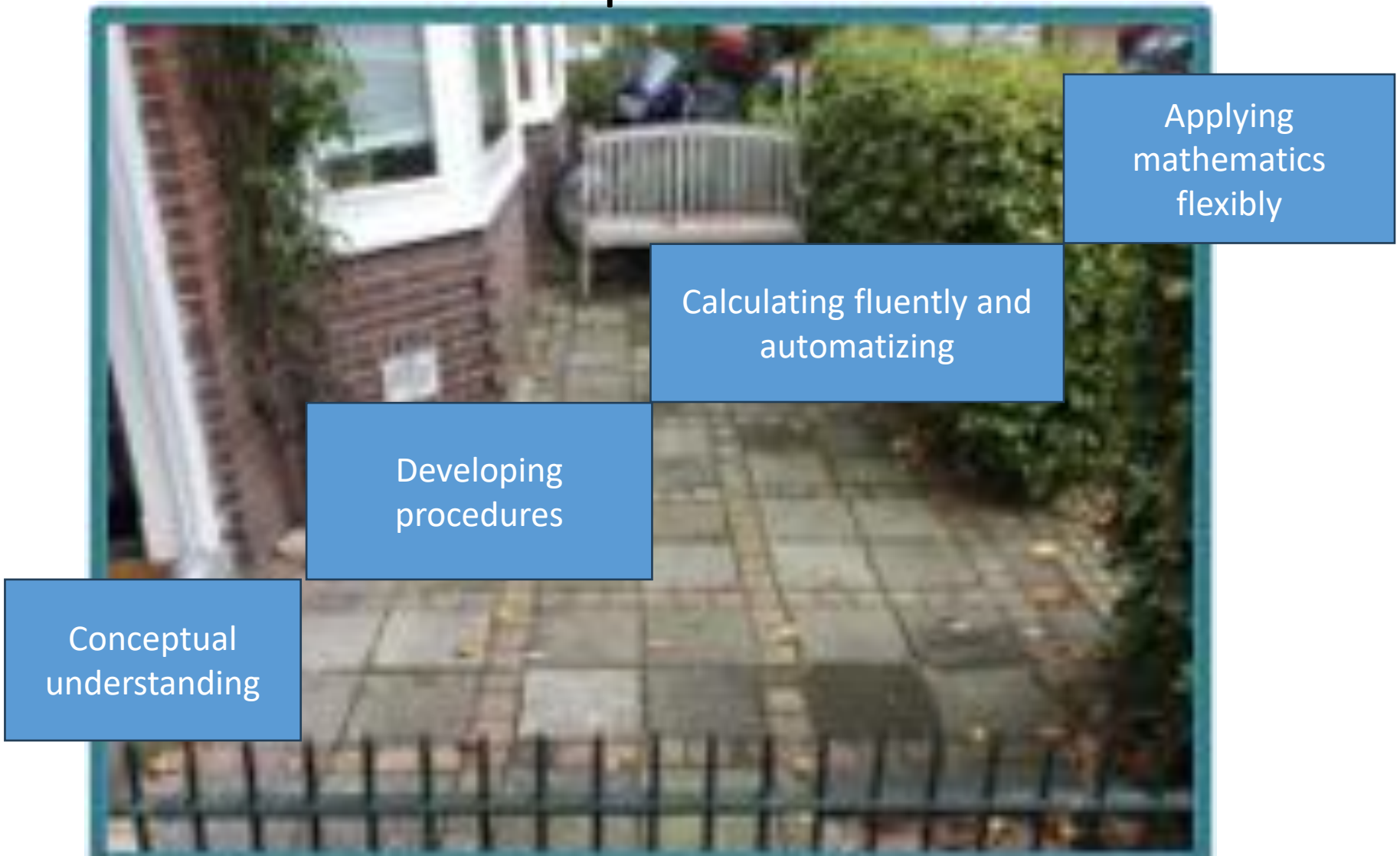
Calculating fluently and automatizing

Developing procedures

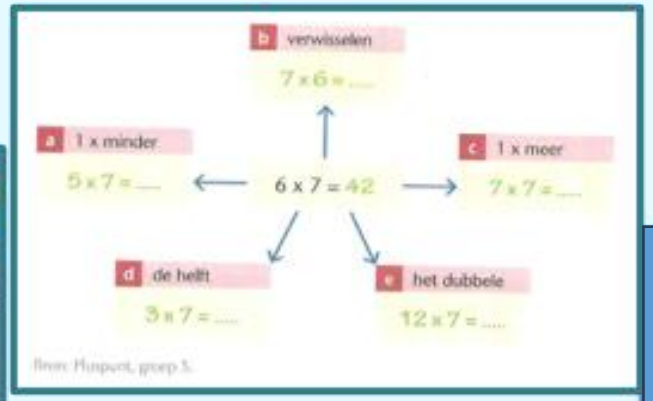
Conceptual understanding



Learning progression in broad steps: Multiplication



Learning progression in broad steps Multiplication



Applying mathematics flexibly

Calculating fluently and automatizing

Developing procedures

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

7	8	9

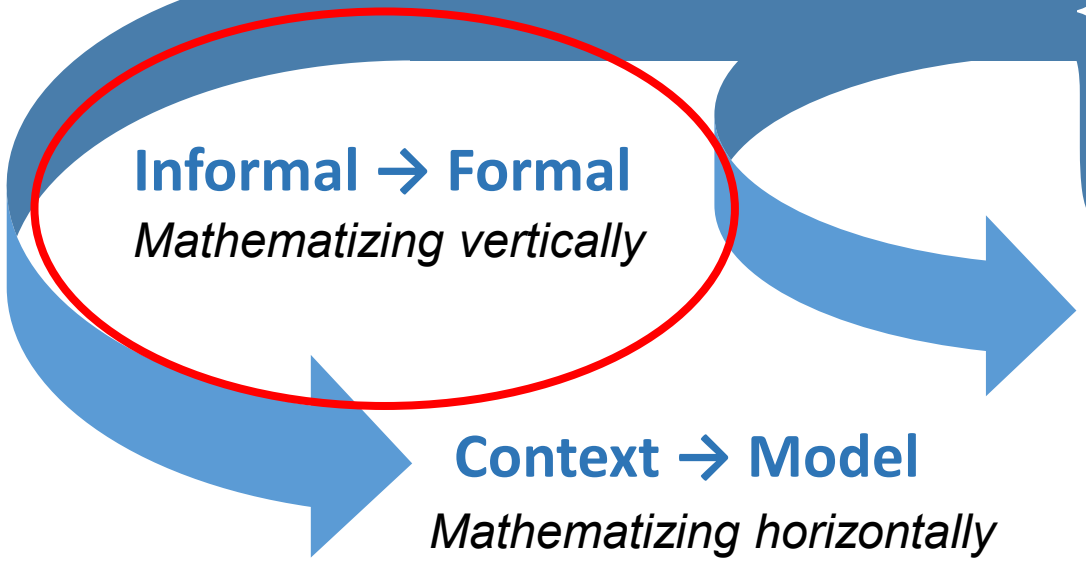
1	2	3	4	5	6

Conceptual understanding

Classroom video?

Mechanistic mathematics education	Realistic Mathematics Education
- Teaching as transmission	- Activity principle
- Bare number problems - Less attention to applications (in any case not at the start)	- Reality principle
- Fixed solution procedures (recipes) - Directly working at the formal level	- Level principle * Various levels of understanding * Progressive schematization * Models as bridges to a more formal level
- Isolated strands/atomized learning	- Intertwinement principle
- Mostly individual seat work	- Interactivity principle
- Direct instruction, step-by-step	- Guidance principle

Realistic Mathematics Education



- **Activity principle**

- **Reality principle**

- **Level principle**

  * Various levels of understanding
  * Progressive schematization

  * Models as bridges to a more formal level

- Intertwinement principle

- Interactivity principle

- Guidance principle

3. Illustrations

Informal → Formal

A scoop holds $\frac{1}{5}$ kg of flour. How many scoops of flour are needed to fill a bag with 6 kg of flour?

Answer: $6 \div \frac{1}{5}$

6×5

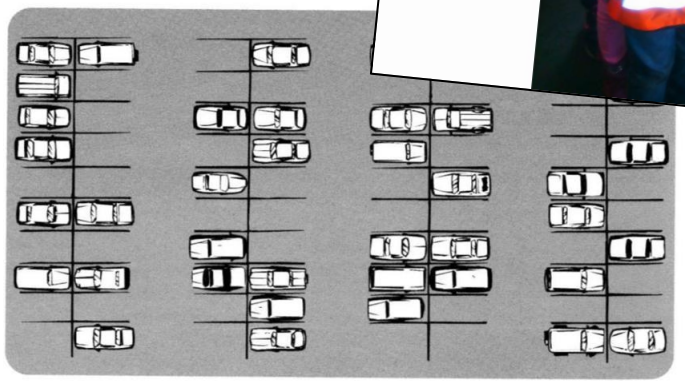
30 scoops

Models as bridges to a more formal level

Context → Model

Bus context

Models as bridge



0 cars 80 cars

Informal → Formal

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Answer: $6 \div \frac{1}{5}$

6×5

30 scoops

Informal → Formal

Formal strategy

$$6 \div \frac{1}{5} = 6 \times \frac{5}{1}$$

	× 5		× 6
number of scoops	1	5	30
kg	$\frac{1}{5}$	1	6
	× 5		× 6

Informal context-connected strategy

1 scoop holds $\frac{1}{5}$ kg;

so, 1 kg is 5 scoops

and 6 kg is 6 times 5, is 30 scoops.

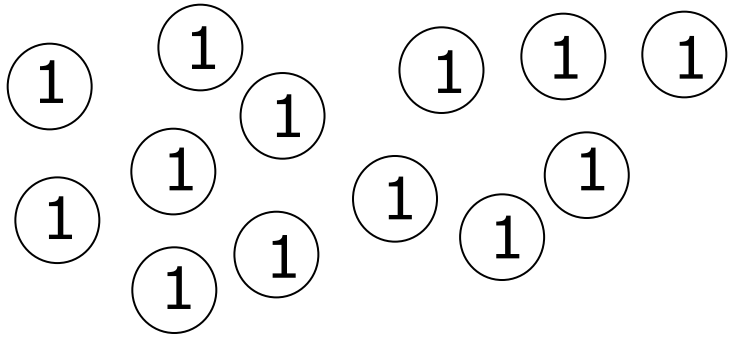
Grade 1 Various levels of understanding

Informal → Formal

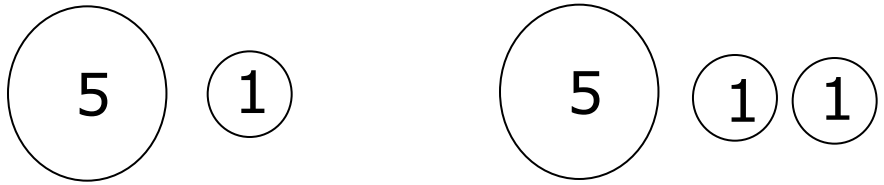
Grade 1 Various levels of understanding

Informal → Formal

Maureen



Thijs and Nick



Luuk *"First, put three euro out of the six to the seven euro; that makes ten euro; and three makes thirteen euro"*

Hannah *"Six and six is twelve; and one makes thirteen euro"*

Grade 1

Various levels of understanding

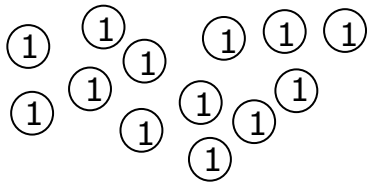
Informal → Formal

six and six is …

formal calculation & number relations

calculation by structuring

cross-section



calculation by counting

Grade 1

Various levels of understanding

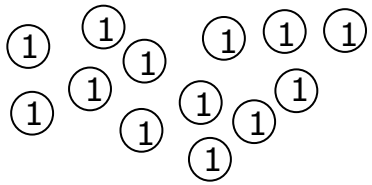
Informal → Formal

six and six is …

formal calculation & number relations

calculation by structuring

cross-section



calculation by counting

calculation by structuring

formal calculation

longitudinal-section

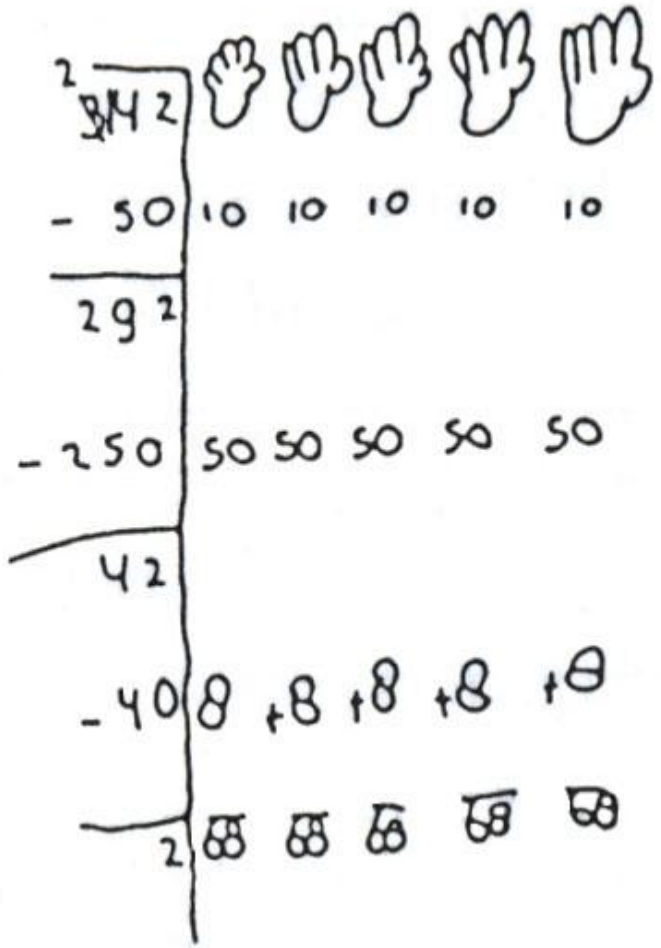
342 match stickers are distributed equally among **5** children.
How many stickers does each of the children get?

Informal → Formal

Progressive complexization

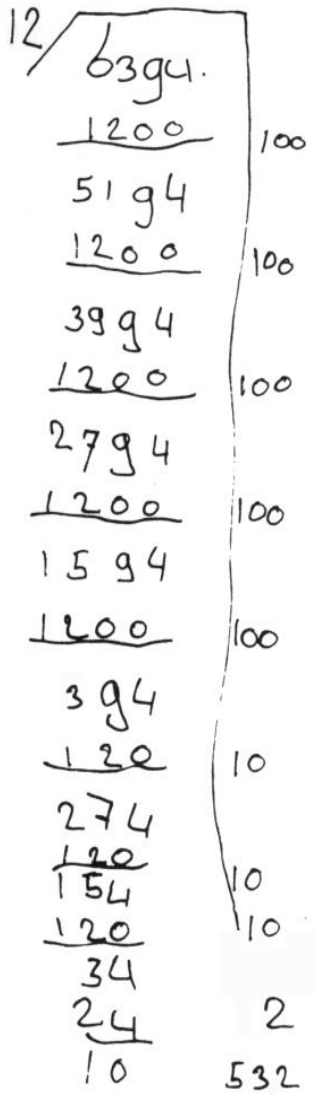
```
2 / 6 \ 3
    6
    0

3 / 72 \ 24
    6
    12
    12
     0

5 / 342 \ 68
    30
     42
     40
      2
```

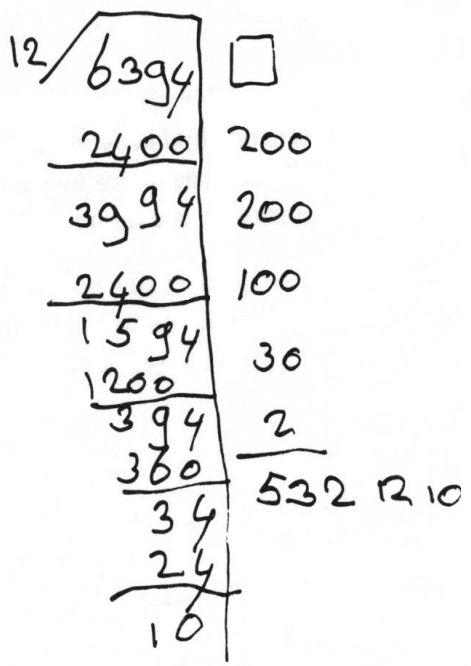
Progressive schematization

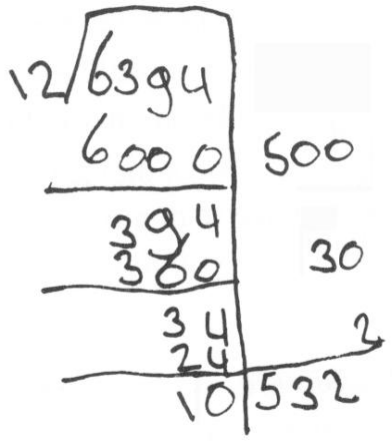


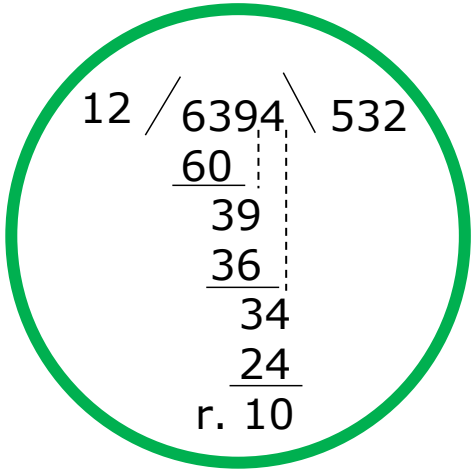
Progressive schematization

Informal → Formal





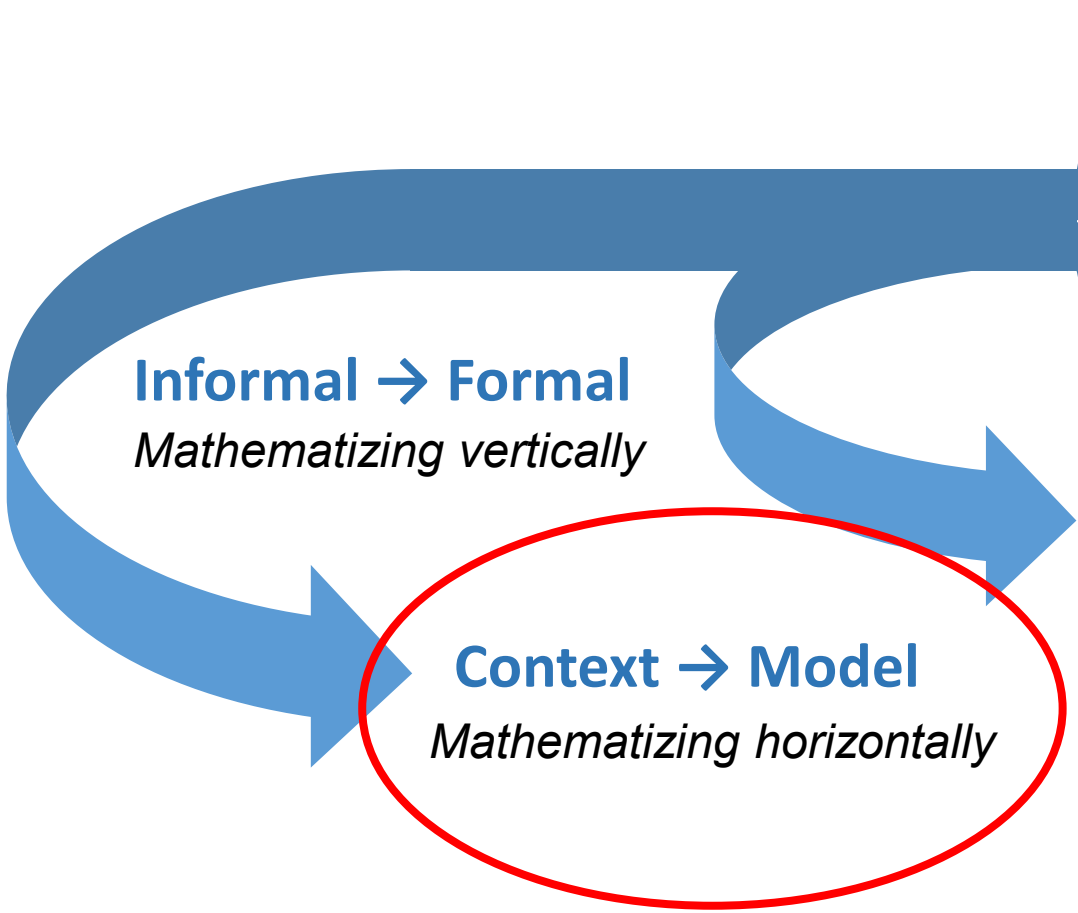




whole-number-based written calculation

digit-based written calculation or algorithmic calculation

Realistic Mathematics Education



- **Activity principle**

- **Reality principle**

- **Level principle**

  * Various levels of understanding
  * Progressive schematization

  * Models as bridges to a more formal level

- Intertwinement principle

- Interactivity principle

- Guidance principle

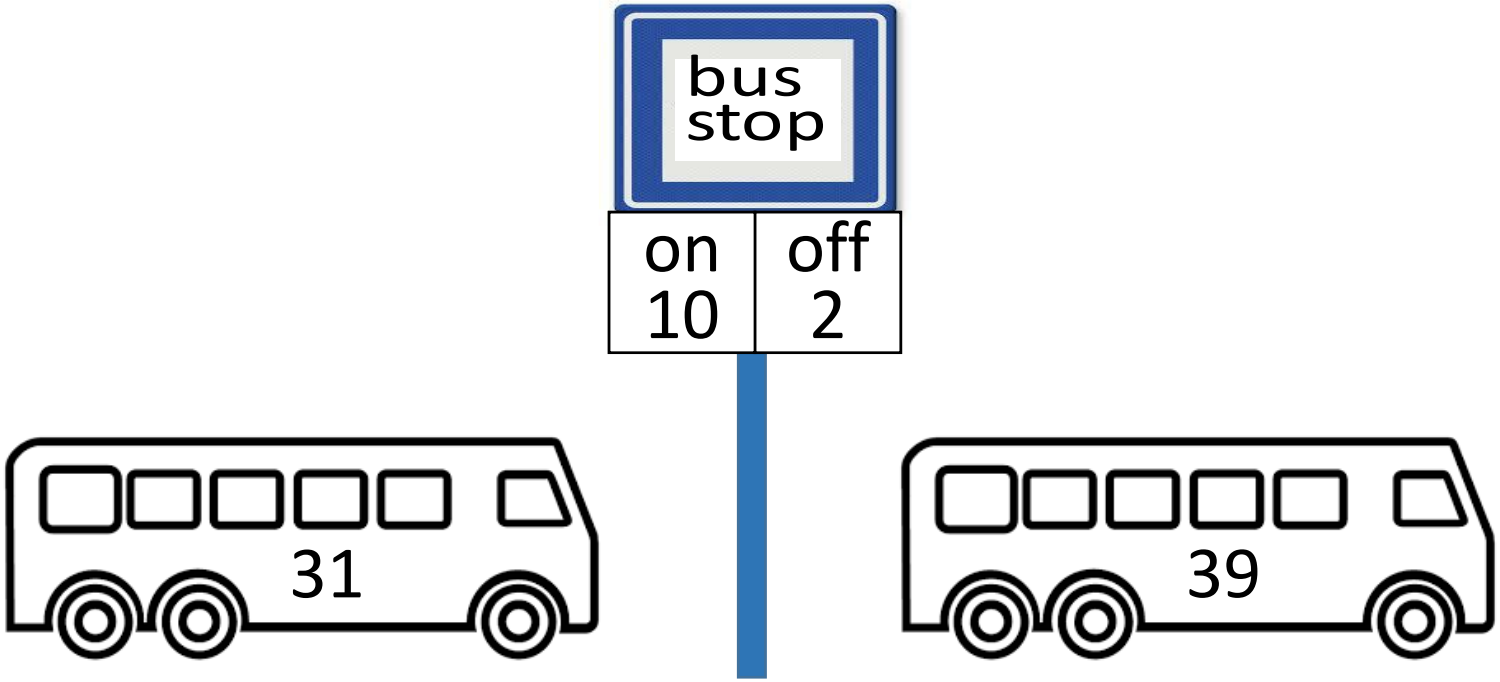
Models as bridges to a more formal level

Context → Model

Bus context

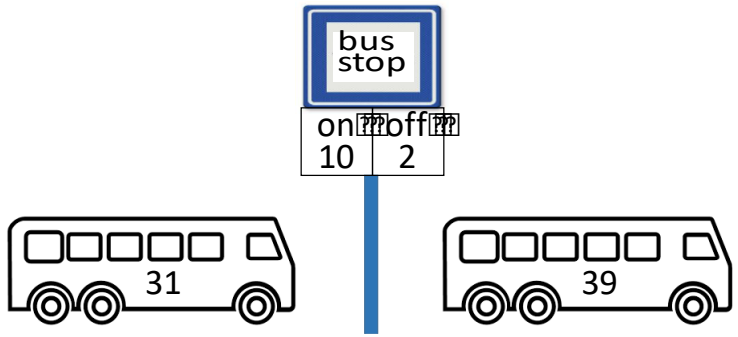
Models as bridges to a more formal level

Context → Model

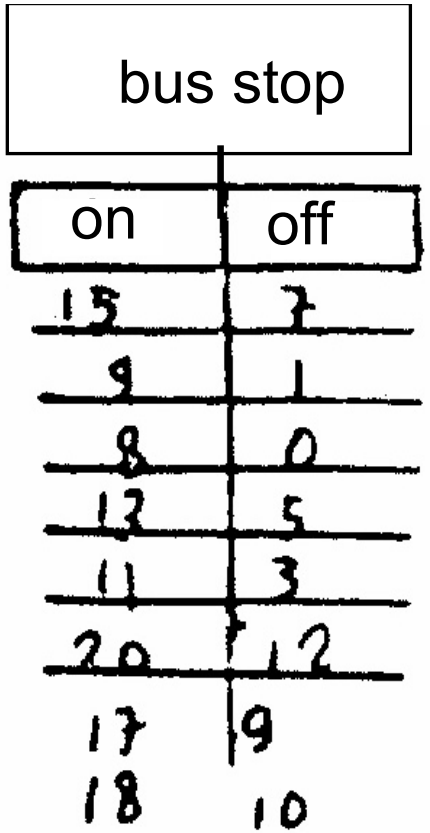


Models as bridges to a more formal level

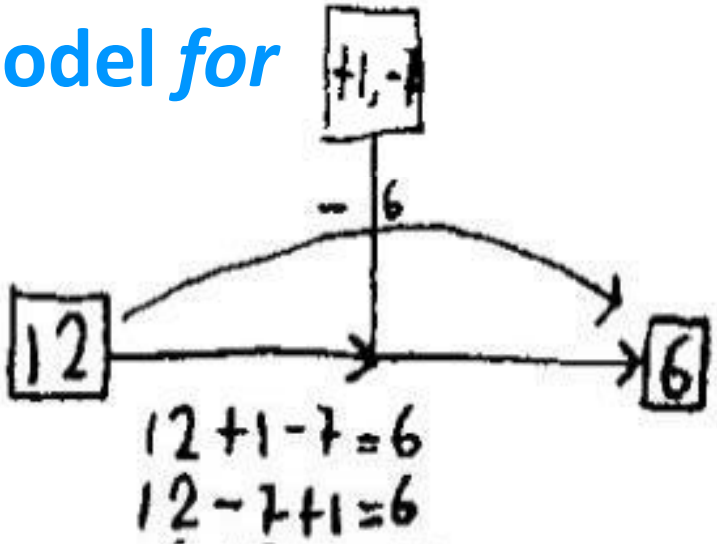
Context → Model



model *of*

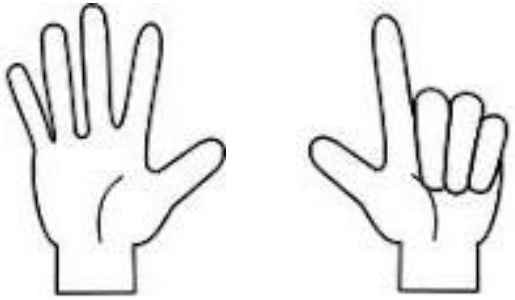


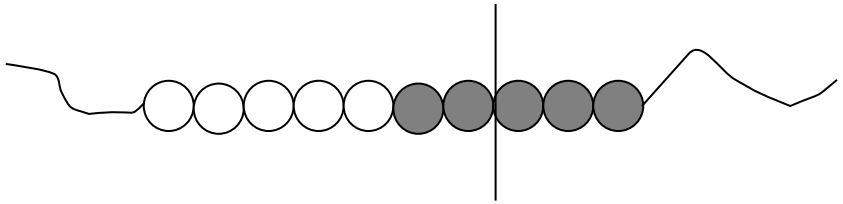
model *for*

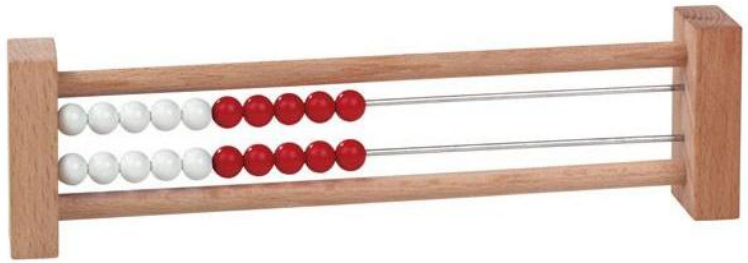


Models as bridges to a more formal level

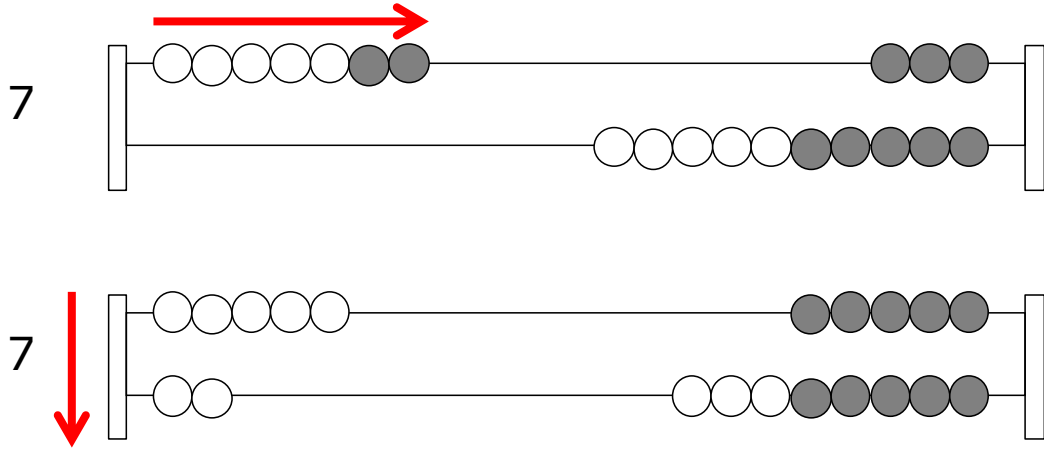
Context → Model







arithmetic rack "rekenrek"

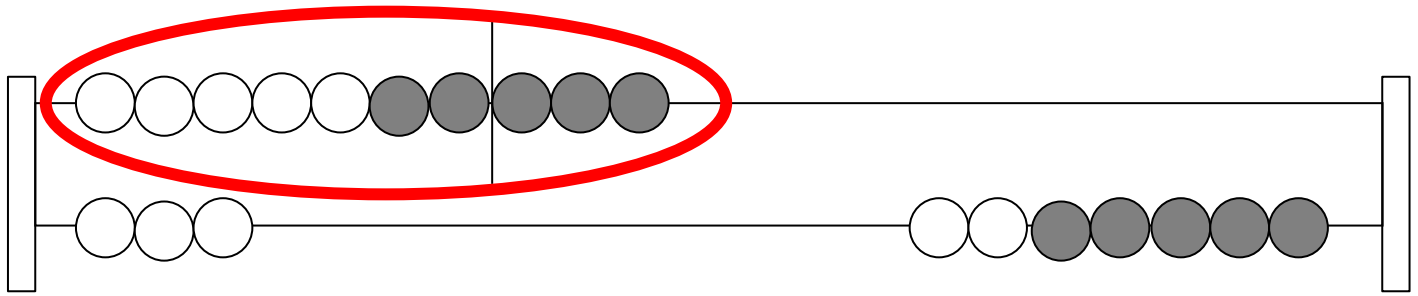


Context → Model

Models as bridges to a more formal level

Context → Model

7+6



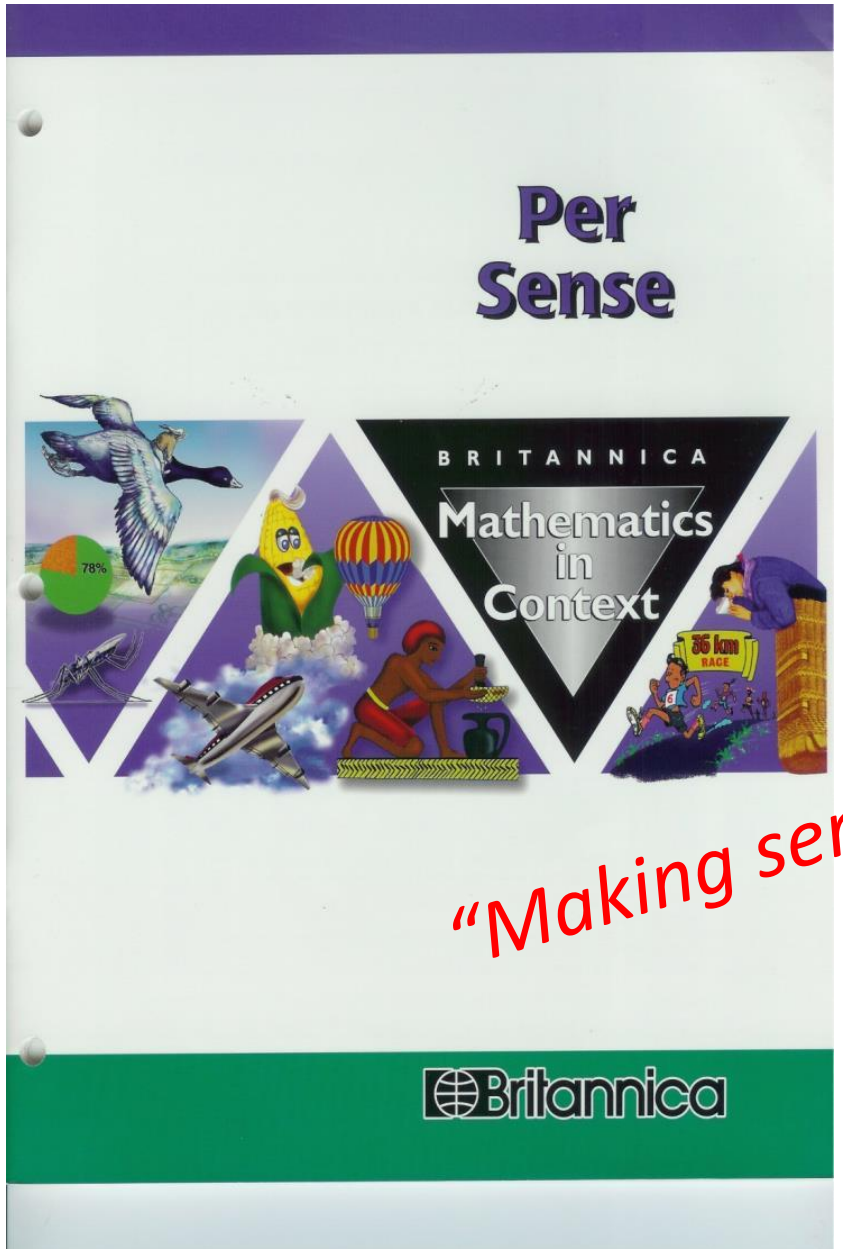
7 and 3 is 10, and then 3 more is 13

7+6

7 is 5 and 2, and 6 is 5 and 1,
5 and 5 is 10, and 2 and 1 is 3, makes 13 in total

Models as bridges to a more formal level

Context → Model



Mathematics in Context

Grade 5

Romberg & De Lange (Eds.)

(1997-1998 ...)

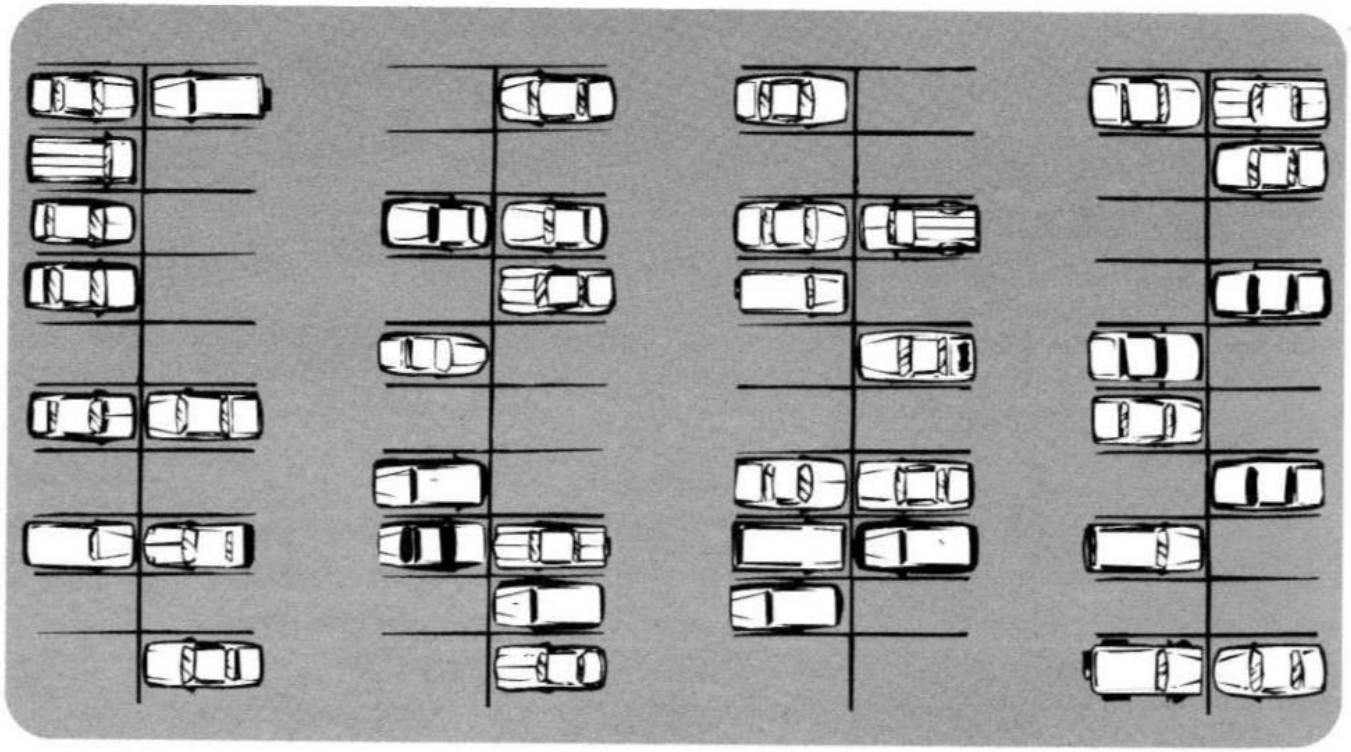
*"Making sense of **percents**"*

Models as bridges to a more formal level

Context → Model

Models as bridges to a more formal level

Context → Model



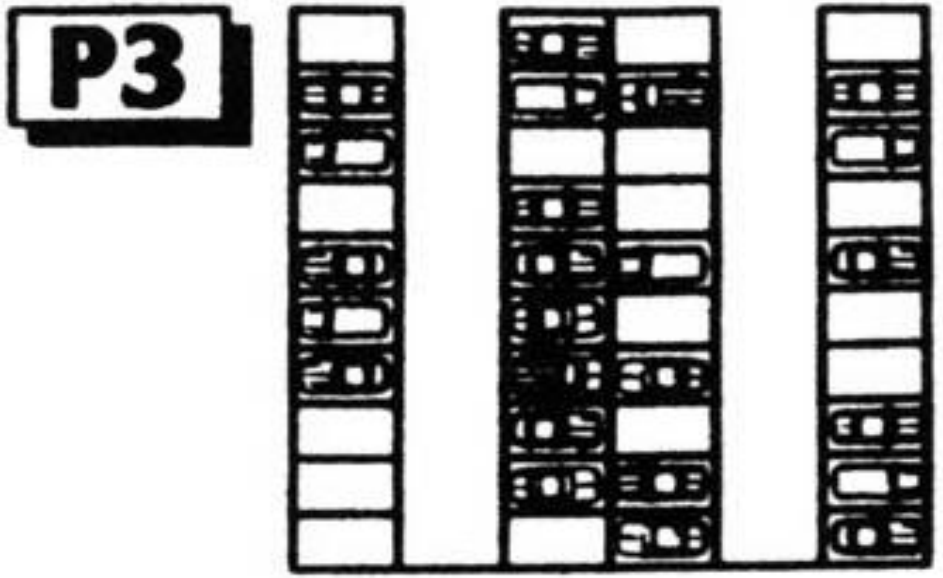
0 cars 80 cars

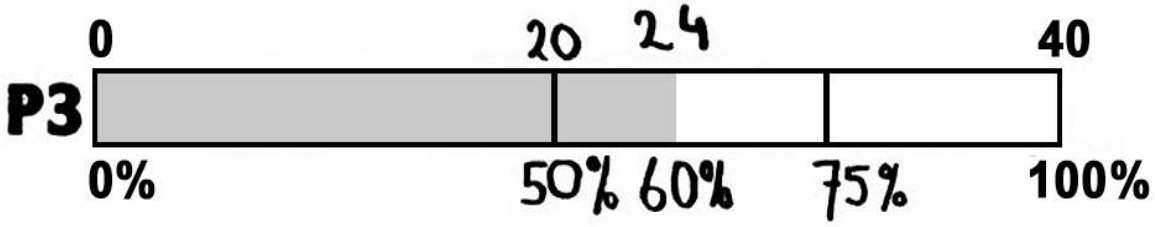
occupation meter

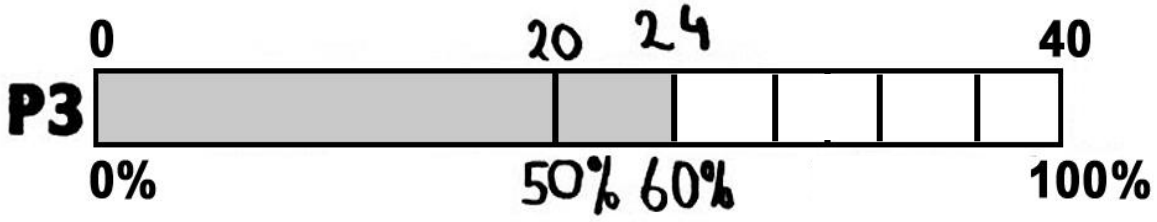
percentage bar

Models as bridges to a more formal level

Context → Model







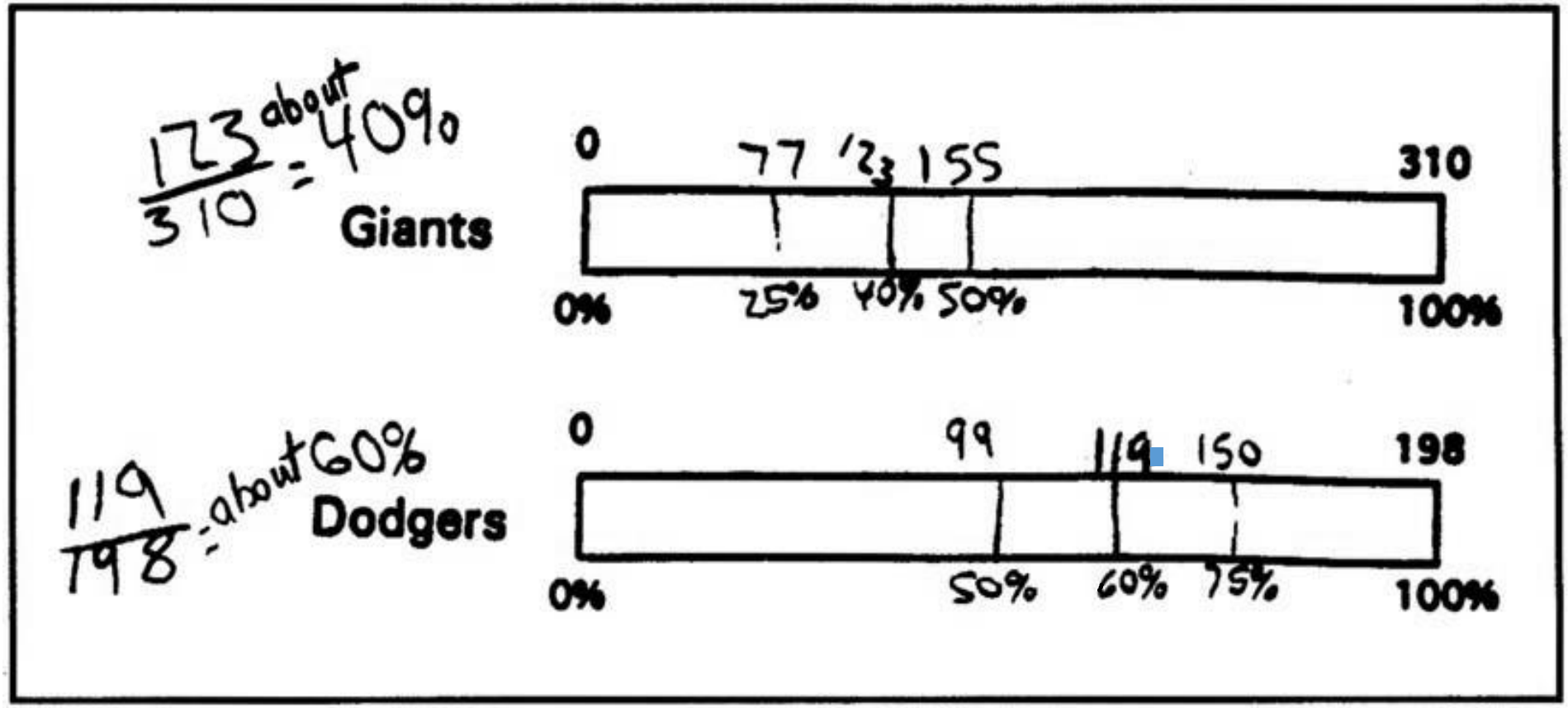
Context → Model

Favorite baseball souvenir

Giants fans (310): 123 vote for cap

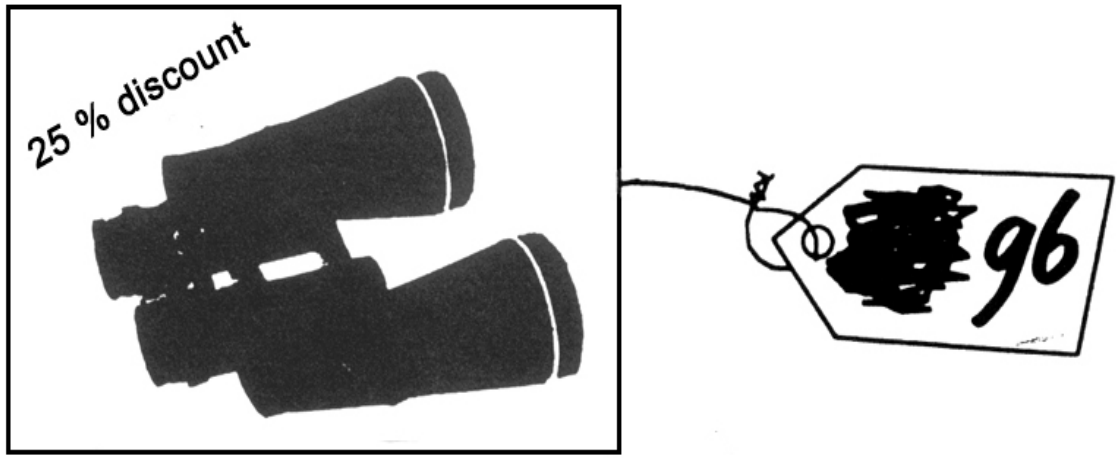
Dodgers fans (198): 119 vote for cap

Which fans like the cap the most?



Models as bridges to a more formal level

Context → Model



What was the regular price?

96.00
+32.00
$128.00

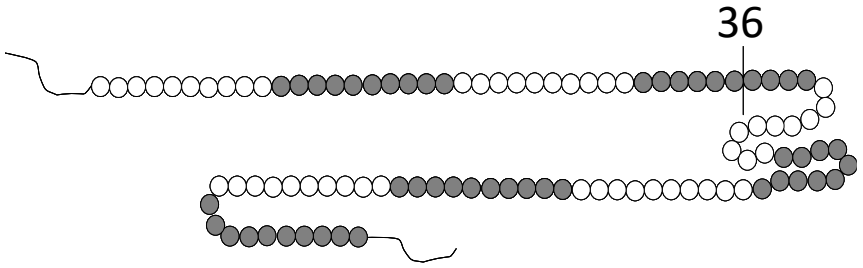
$96.00 $128.00

$96.00 is 75%. 75% is 3 of 4. So if you divide $96.00 by 3 you get $32.00 then you add.

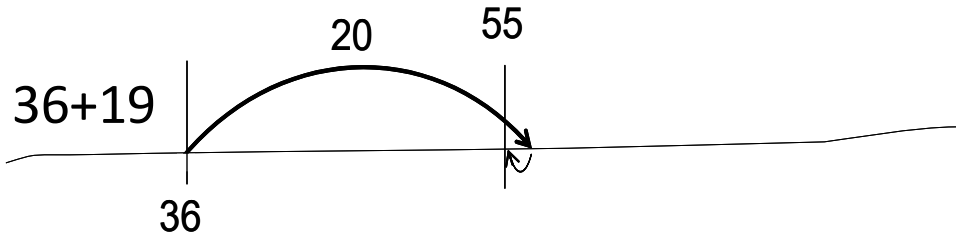
25% 50% 75% 100

COHERENCE OF MODELS

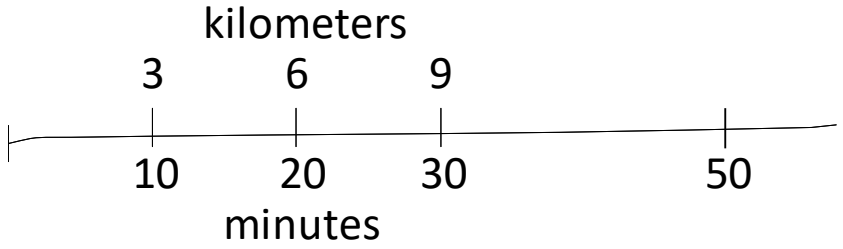
Context → Model



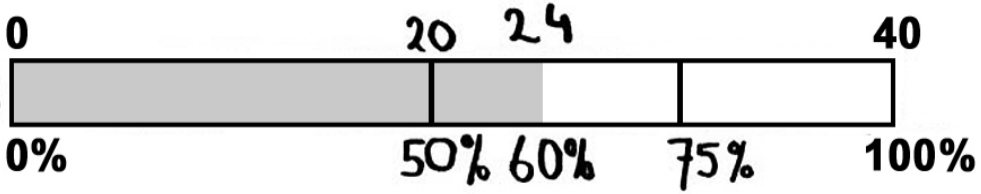
chain of beads



empty number line



double number line



percentage bar

kilometers	3	6	9		5	
minutes	10	20	30	50		

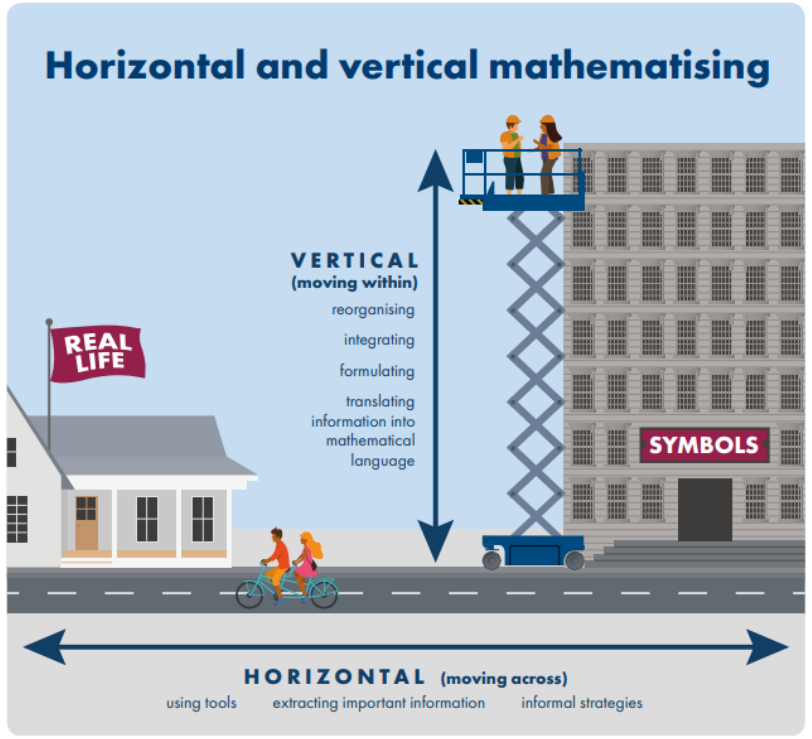
ratio table

Realistic Mathematics Education

- Activity principle
- Reality principle
- Level principle
  - * Various levels of understanding
  - * Progressive schematization
  - * Models as bridges to a more formal level
- Intertwinement principle
- Interactivity principle
- Guidance principle

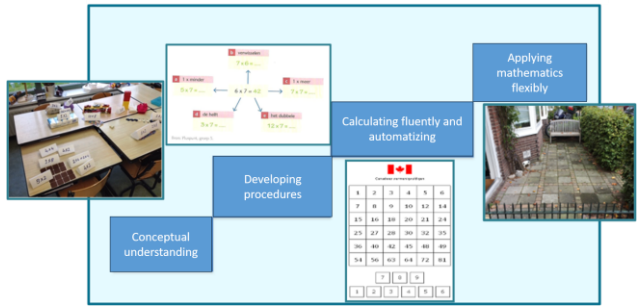
Informal → Formal
Mathematizing vertically

Context → Model
Mathematizing horizontally



Adapted from Treffers (1978), Freudenthal (1991) & Barnes (2005)

Learning progression in broad steps
Multiplication



4. Next lecture: Big Mathematics Day

- See you in our next lecture: **October 7th**
- On the translation of these principles into the Big Mathematics Day
  - Doing mathematics in coherent activities in and outside the classroom
- With examples of such activities of different Big Mathematics Days in the Netherlands