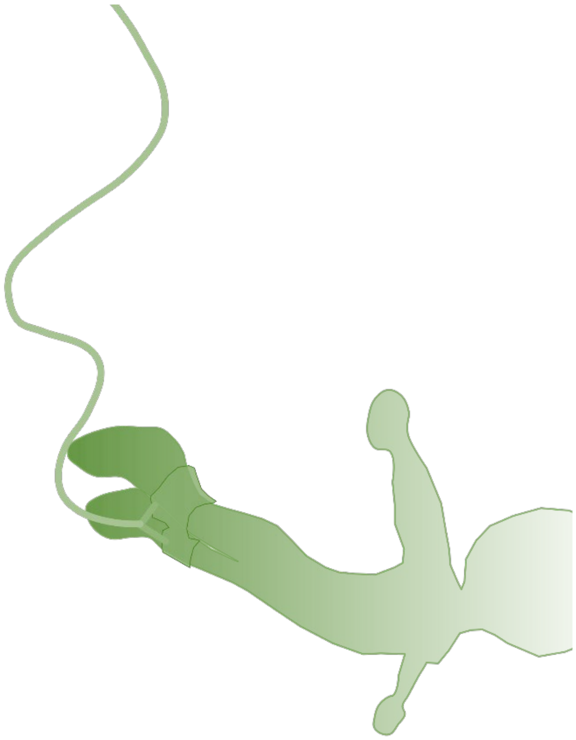

ENERGY CONSERVATION

Energy in diagrams

"If you want to find the secrets of the universe, think in terms of energy, frequency and vibration."

Nikola Tesla

The module "Energy conservation – energy in diagrams" is developed as part of a PhD research into understanding problems of secondary school students with regard to the domain 'Quantumwereld'. It is intended as preparation for Domain F1 Quantum World of the Dutch secondary school curriculum, in particular for the understanding of quantum phenomena in the context of trapped particles and tunneling.

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1. INTRODUCTION

1.1 WORK AND ENERGY

In daily life, we often talk about work and energy when something takes effort or effort. In physics, the concepts of work and energy are very important, they give more insight into movements and movement changes.

Work is the energy change of an object caused by a force that acts on that object. This energy change causes a change in the position or velocity of the object, or the formation of heat. The work depends on the size of the force and the distance over which the force is exerted. If the force is applied in the same direction as the movement of the object, the work can be calculated using the following formula:

$$W = F \cdot s \tag{1}$$

W is the work in J
F is the force in N
s is the distance travelled in m

Kinetic energy is the energy related to the movement of an object; only objects that move have kinetic energy. Thus, an object at rest has no kinetic energy. The kinetic energy depends on the mass and velocity of the object. The kinetic energy can be calculated with the following formula:

$$E_k = \frac{1}{2} \cdot m \cdot v^2 \tag{2}$$

E_k is the kinetic energy in J
m is the mass in kg
v is the velocity in m/s

Potential energy is energy stored within a system, energy that can be converted into other forms of energy. Potential energy expresses how much work can be performed on an object by the forces working on the object. The potential energy therefore belongs to the object, but cannot be seen separately from the cause of the force that acts on the object. You have already come across several examples of potential energy. For example, the **gravitational energy** of an object at a certain height above the ground indicates how much work gravity can perform on the object. The **elastic energy** of a mass on a pressed spring indicates the work which the spring can perform on the mass. **Electrical energy** is also an example of potential energy. All these examples of potential energy will be discussed during this module.

1.2 Law of conservation of energy

An object can possess two types of energy, kinetic energy and potential energy. These two energies together are also called the **total energy** of the object. The **law of conservation of energy** states that energy can never be lost. If an object loses potential energy, it is converted into other forms of energy, and the object will gain kinetic energy. In real life, part of the lost potential energy will be converted into another form of energy: heat. This heat is not energy of the object itself, this energy ends up in the environment of the object. Because energy is never lost, the total energy of a **system**, that is, an object (or combination of objects) *and* the environment, always remains the same. However, the energy of an object within a system may change, as illustrated in the following example:

Example: Energy changes within a system

An object within a system can get more or less total energy; consider a car on a flat road that first pulls up and then slows down. During acceleration, the velocity and kinetic energy increase and during braking the velocity and kinetic energy decrease again.

During acceleration, the car's engine exerts a force on the car and performs work, causing the car to accelerate and gain more energy. The engine obtains the necessary energy for this from the fuel. This fuel contains chemical energy, which is also a form of potential energy. So the car gets more energy while accelerating, but the environment, in this case the engine with the fuel, gets less energy. Wo, the total energy of the car *and* the environment remains the same.

The car decelerates because a friction force is exerted on the car, for example due to the friction between the tires and the road. This friction creates heat; the tires and the road surface are warmed up. Then, when the car's tires then cool down again, they transfer their heat to the air around them. In physics, heat is also a form of energy. So, during braking, the car receives less total energy, but the environment, in this case the road surface and the air, receive more energy in the form of heat. Thus the total energy of the car *and* the surrounding area together is preserved.

This example shows that when applying the law of conservation of energy, it is important to properly distinguish between the object, or combination of objects, and the environment. Thus, the total energy of an object can increase, but then the energy of the environment will decrease or vice versa. The total energy of an object, or combination of objects, is preserved only if no work is done on the object by the environment and if no work is done by friction forces.

2. Earth's gravitation

2.1 Gravitational force and energy on earth

Objects are attracted by the Earth. The force that the Earth exerts on objects is called **gravity.** As long as the object is at a small distance from the earth's surface, this force may be considered constant. In that case, gravity is expressed as:

$$F_z = m \cdot g \tag{3}$$

F_z is the gravity on the object in N
m is the mass of the object in kg
g is the gravitational acceleration in m/s^2 (on Earth about $9.81\ m/s^2$)

Because of gravity, it takes effort to lift an object. In physics, the effort you have to make is called the work you perform on the object. This work increases the energy of the object; the object gains **Earths gravitational energy**. The Earths' gravitational energy is expressed as:

$$E_z = m \cdot g \cdot h \tag{4}$$

E_z is Earths gravitational energy in J
m is the mass of the object in kg
g is the gravitational acceleration in m/s^2 (on Earth about $9.81\ m/s^2$)
h is the height at which the object is located in m

The height h is usually defined on the earth's surface as h = 0. Later we will see that this can also be chosen differently. For now, we define h = 0 on the earth's surface, unless otherwise indicated.

Exercise 1

a. Using formulas (1) and (3), calculate the work you need to do to lift a bag with a mass of 5.0 kg from the ground at a constant velocity unto a height of 2.0 m.
b. Using formula (4) calculate the gravitational energy of a bag with a mass of 5.0 kg at a height of 2.0 m.
c. Explain why the outcomes of questions a and b are the same.

2.2 Potential energy

Earths gravitational energy is potential energy; it is a measure of the work that gravity can perform on an object. If you hold an object at a certain height and then release it, the object will fall. The gravity that acts on the object causes the object to accelerate. As a result, the object receives more and more velocity and more kinetic energy. Because gravity performs work, the potential energy (gravitational energy) is converted into kinetic energy.

If we disregard the friction on the object during the fall, the total energy of the object remains the same during the fall. Work is done, but only by the force that causes the interaction between E_k and E_z, i.e. gravity. Since no other forces work on the object, the total energy of the object remains constant.

EXAMPLE: THE HEIGHT OF A BALL

A soccer ball with a mass of 0.5 kg is kicked from the ground perpendicularly upward. The kick gives the ball velocity and kinetic energy. We do not take into account the frictional forces on the ball.

When the ball moves upwards, the potential energy will increase. As a result, the kinetic energy decreases and, because the ball moves perpendicularly upward in this situation, the velocity eventually becomes 0 m/s. At this point, the ball reaches its maximum height. This can be seen in Figure 1: at a height of 3.25 m, the kinetic energy is 0 J. The graph of E_t and E_p intersect in this point. Since the total energy cannot be increased, E_p can no longer increase from this moment on and the ball cannot exceed 3.25 m.

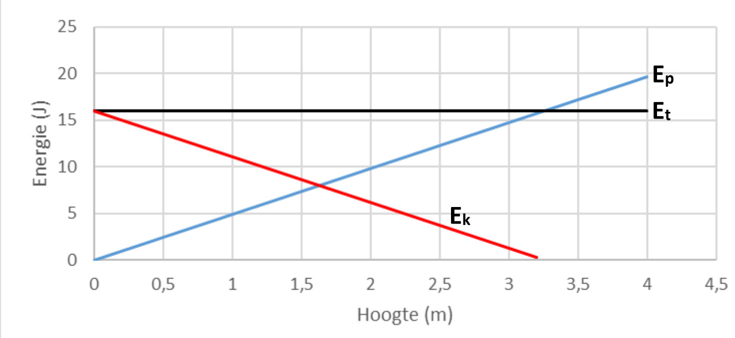

FIGURE 1: ENERGY DIAGRAM OF A FOOTBALL UNDER THE INFLUENCE OF GRAVITY

EXERCISE 2

a. Explain what force(s) in the example above cause the potential energy of the ball.
b. Explain how you can see in the energy diagram that friction forces are not taken into account.
c. Explain what happens to E_k, E_t and E_p after the ball reaches its highest point.

EXERCISE 3

The football from the example jug also in other directions be kicked away. The direction can be expressed in an angle ϑ relative to the earth's surface, see figure 2.

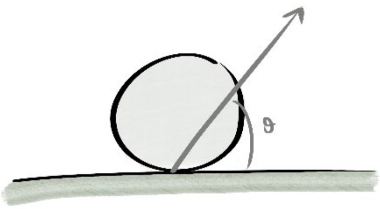

FIGURE 2 THE ANGLE UNDER WHICH A FOOTBALL CAN BE KICKED AWAY

a. Use an energy reflection to explain what the direction of the ball should be to reach the maximum height.
b. Explain that the size of the velocity of the ball at 1 m height does NOT depend on the direction ϑ,in which the ball is kicked.

2.3 ROLLER COASTERS

A nice illustration of the interaction between kinetic energy and gravitational energy is a roller coaster (Figure 3). In a roller coaster, you are first raised to the top of the first hill, giving the train a large amount of gravitational energy. At the top of the hill, the car is released and from that moment on, the car goes down without propulsion. During this process, the gravitational energy is converted into kinetic energy. During a ride on the roller coaster, you often go up and down several times, so that due to the change in potential energy the velocity changes every time. When we disregard friction, the total energy during the ride remains constant.

FIGURE 3 A ROLLERCOASTER

EXERCISE 4

Gary designed a roller coaster. His design is shown in Figure 4. In this roller coaster, the car is released on the first hill (marked with 'start'), with an initial velocity of $v = 0$ m/s. Use an energy reflection to explain why this is not a good design.

EXERCISE 5

Figure 5 on the next page shows a height-place diagram of a rollercoaster ride. Between $x = 0$ and $x = 11$ m the car is lifted, then the ride starts and the car is released. The car (including passengers) has a mass of 1000 kg. Friction forces are disregarded.

a. On your worksheet, draw a diagram in which you plot the car's potential energy (E_p) against the position.
b. Determine the total energy (E_t) of the car and add this to the diagram of a.
c. Use the energy conservation law to determine the kinetic energy (E_k) of the train during the journey. Add this to the diagram of a.
d. Use the drawn graphs to determine the velocity at $x = 30$ m and $x = 100$ m.

FIGURE 4 GARY'S DESIGN OF A ROLLER COASTER

EXERCISE 6

We consider once again the rollercoaster ride of figure 5. Now we will not leave the friction out of the equation. The energy of the car that is converted into heat by friction can be calculated using the following formula:

$$Q = F_w \cdot s \tag{5}$$

Q is the heat in J
F_w is the friction force in N
s is the distance travelled in m

a. Using diagram 5, determine the maximum level of average friction force to get just on top of the first hill (after the start).
b. Using diagram 5, determine the maximum average friction force required to be able to finish the entire ride. To do this, first estimate the length of the track.

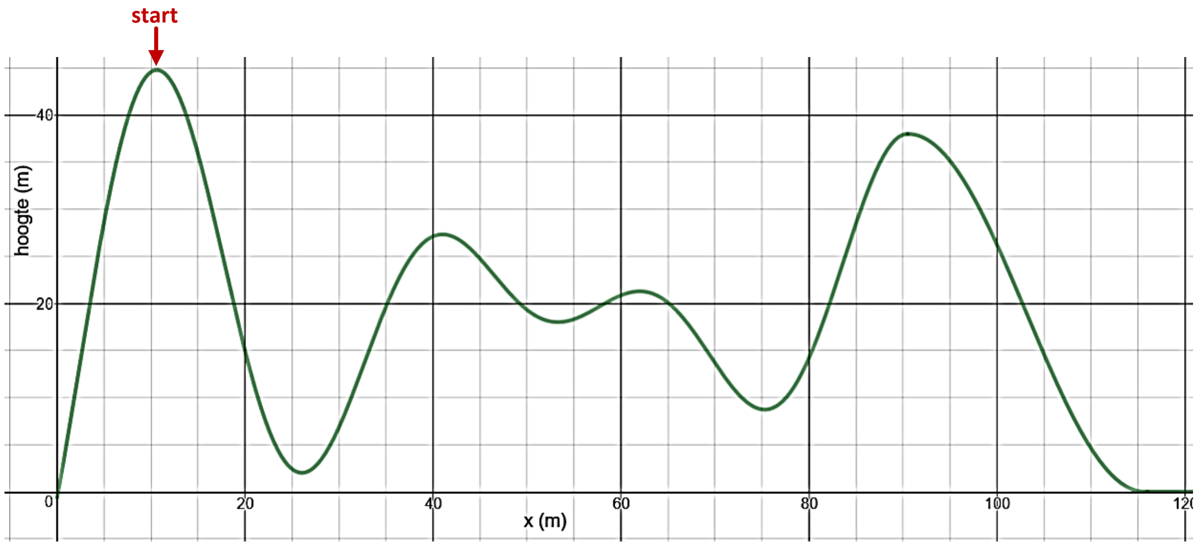

FIGURE 5 THE HEIGHT-POSITION DIAGRAM OF A ROLLERCOASTER RIDE

3. ELASTIC ENERGY

3.1 ELASTIC FORCE AND ENERGY

When an object is attached to the end of a spring and the spring is pressed or stretched, the spring exerts a force on the object. This force is called **elastic force.** You have learned in previous chapters that resilience has been given by:

$$F_s = -C \cdot u \tag{6}$$

F_s is the elastic force in N
C is the spring constant in N/m
u is the displacement of the spring in m

Due to the elastic force, it takes effort to compress or stretch a spring; work needs to be performed on the spring. This increases the energy of the spring; the spring gains **elastic energy**. The spring energy is given by:

$$E_s = \frac{1}{2} \cdot C \cdot u^2 \tag{7}$$

E_s is the elastic energy in J
C is the spring constant in N/m
u is the displacement of the spring in m

3.2 POTENTIAL ENERGY

Elastic energy is potential energy. When an object is attached and held at the end of a pressed or stretched spring, the object will move when it is released. The force of the spring acting on the object causes the object to accelerate. This gives the object a higher velocity and more kinetic energy. The force of the spring performs work, converting potential energy (elastic energy) into kinetic energy.

When gravity is not considered, a mass-spring system has u = 0 as its **equilibrium position**. In that state, the spring is not stretched or compressed, so the spring does not exert force on the object attached to it. There is therefore no (resulting) force that can perform work on the system and therefore the spring energy in this point is 0 J. This corresponds to the physical principle that an object will always occupy the position with the lowest possible potential energy.

When gravity is taken into account, the total potential energy ($E_p = E_z + E_v$). As you can see in Figure 6, this changes the location of the minimum of the potential energy. From this it can be concluded that the equilibrium position has been moved to a different position.

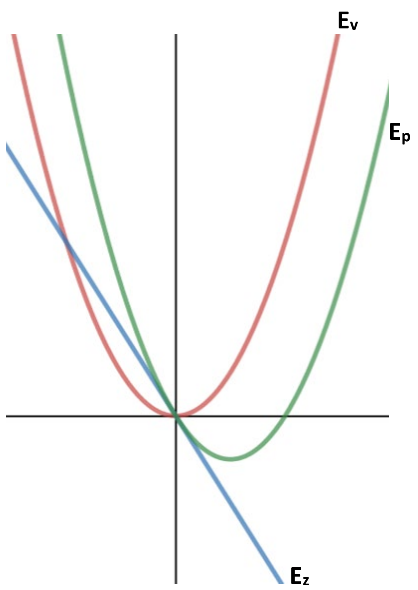

FIGURE 6 THE POTENTIAL ENERGY OF A MASS SPRING SYSTEM IN WHICH GRAVITY IS TAKEN INTO ACCOUNT

EXAMPLE: A MASS-SPRING SYSTEM

Consider a mass-spring system (see Figure 7). The spring is stretched by 10 cm by pulling the mass ($m = 50$g); work is done by muscle strength. Then the block is released. Gravity and friction are not taken into account.

The energy diagram for this situation is shown in figure 8. In the equilibrium position of the spring, the displacement $u = 0$ cm and the spring is neither stretched nor compressed. If the spring is stretched, the u is positive. The diagram shows the total energy E_t, kinetic energy E_k and the potential energy E_p. In this diagram you can see that the maximum displacement of the mass depends on the total energy and the potential energy of the system. At the point where $E_t = E_p$ the kinetic energy is zero, and therefore the mass cannot go beyond this point. When $x \neq 0$ the spring exerts a force towards the equilibrium position. In the energy diagram this can be deduced from the potential energy; a force works in the direction in which E_p becomes smaller (we come back to this in Chapter 5).

FIGURE 7 A MASS SPRING SYSTEM

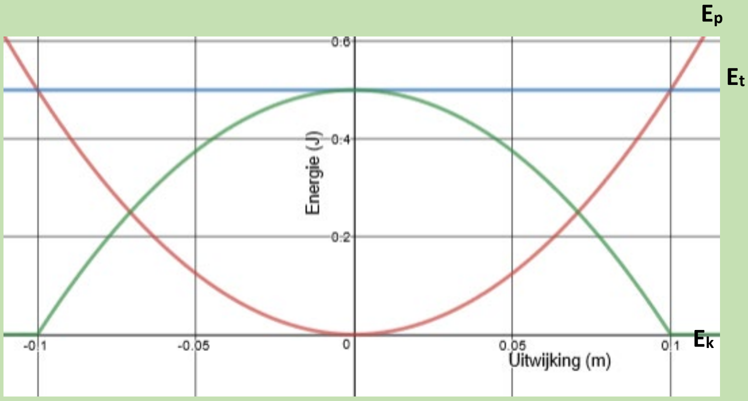

FIGURE 8 ENERGY DIAGRAM OF A MASS SPRING SYSTEM

EXERCISE 7

Answer the following questions for the mass-spring system of Figure 7:

a. Explain what causes the potential energy in Figure 8.
b. Explain why the displacement cannot exceed $x = 0.1$ m.
c. Determine the maximum velocity of the mass and indicate at which position this velocity is reached.
d. Determine the velocity of the block if the spring is stretched for 5.0 cm.

You let the block mass with your hand. As a result, the total energy of the system is halved.

e. In the energy diagram on the worksheet, draw the potential energy, kinetic energy and total energy of the mass block after the collision with your hand.

3.3 BUNGEE RUN

We're looking at a new craze, bungee run. Here, two participants each wear a harness that is attached to an elastic band. The other end of the elastic band that is attached to the back wall of the inflatable bungee run. The participants must run forward as far as possible and place a marker to show the distance they travelled before being pulled back by the elastic band. See Figure 9.

FIGURE 9 BUNGEERUN

(by Ross M Karchner from Mclean, VA, USA - Bungee Run!, CC BY-SA 2.0, https://commons.wikimedia.org/w/index.php?curid=22258391)

EXERCISE 8

The track is 15 m Long. At the beginning of the track, the elastic band is not yet stretched. From $x = 7.0$ m, the elastic band starts to exert a force. We assume that from then on the elastic band will behave like a spring with a spring constant of 100 N/m. Caren, a good sprinter, who has a mass of 65 kg, starts from a standstill and reaches a velocity of 7.0 m/s at $x = 7.0$ m. Consider the acceleration to be constant. Then she stops to run and slides further down the track. Friction is disregarded. When Caren has come to a standstill, she holds on tight to the sides of the track and stands still.

Caren's movement until $x = 7$ m is shown in Figure 10.

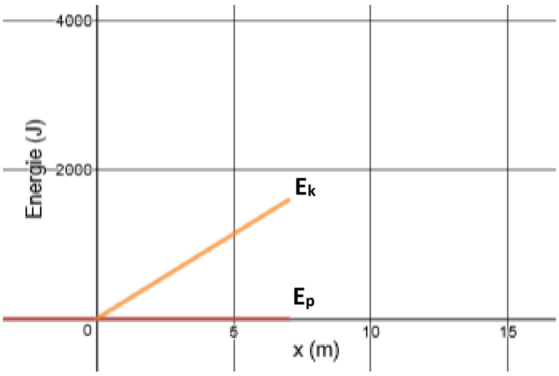

FIGURE 10 ENERGY DIAGRAM OF A BUNGEERUN

a. Complete Figure 10 on your worksheet up to $x = 15$ m.
b. Explain how you can see from the diagram drawn at a. that a force is acting on Caren in the direction of $x = 0$ m.
c. In the same Figure, draw the graph for E_t.
d. Use the graphs to determine how far Caren has come in the end.
e. Determine Caren's velocity at $x = 10$ m.
f. Determine what velocity Caren would need at $x = 7.0$ m to reach the end of the track.

Caren now lets go of the sides of the track and slides back to the starting point. Friction is still disregarded.

g. Using the energy diagram drawn at a. explain how you can see that Caren will slide back to the starting point.
h. Determine at what velocity Caren now collides with the beginning of the bungee run track.
i. In the Figure on your worksheet, draw the diagram of E_p, E_t, and E_k plotted against the position x for this movement.

3.4 BUNGEE TRAMPOLINE (BASED ON THE EXAM: VWO NATUURKUNDE 12, 2011-1, ASSIGNMENT 4)

Another ride that uses spring energy is the bungee trampoline (See Figure 11). In this ride one wears a harness, to which two elastic cords are attached. The elastic cords are attached to steel cables. These cables are wound around a reel by an electric motor. As a result, you are slowly pulled up vertically until you are hanging still high above the trampoline.

FIGURE 11 BUNGEE TRAMPOLINE

By David Hawgood / *Trampoline with assisted bounce, Brighton beach* / CC BY-SA 2.0.

EXERCISE 9

Lars takes a ride on a bungee trampoline. He is given a harness and is attached to the two elastic cords. Each elastic cord has a spring constant of 120 N/m and is stretched 3.1 m from its relaxed position. Lars' center of gravity goes up 2.3 m. Lars's mass (including his harness) is 48 kg.

a. Calculate the work that the electric motor must perform for this purpose.

Lars is then pulled down by a helper until his feet push the trampoline in a bit and he can push off. After a number of swings, Lars makes high, vertical jumps. He's not going to get above the frame. Lars's jumps are simulated in a model. Frictional forces are not taken into account in this model, nor Lars' muscle power. This provides the energy diagram of Figure 12 .

This figure shows different energies as a function of time:

- kinetic energy E_k
- gravitational energy E_z
- spring energy of the elastic cords $E_{s\text{-}el}$
- spring energy of the trampoline $E_{s\text{-}tr}$
- total energy E_t

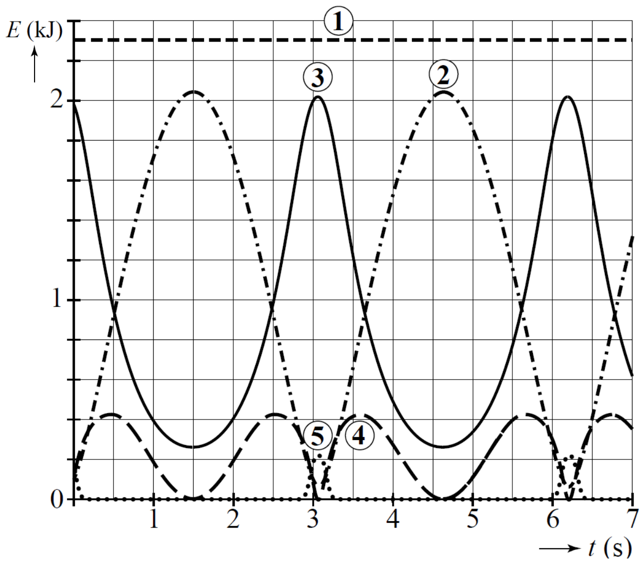

FIGURE 12 DIFFERENT ENERGIES PLOTTED AGAINST TIME WHEN JUMPING WITH A BUNGEE TRAMPOLINE

From the exam: Natuurkunde 12 VWO, 2011-1 1, figure 3.

b. For each number in Figure 12, indicate which of the above energies it is.

c. Use diagram 12 to determine Lars' maximum velocity and indicate the height at which he reaches this velocity.

d. Use diagram 12 to determine the maximum height lars reaches.

EXCERCISE 10

We consider Lars' jump with the bungee trampoline again. Jesse creates an *energy-height* diagram based on figure 12, this diagram is shown in Figure 13. When Lars is at the lowest point $h = 0$ m.

a. For each number in Figure 13, indicate which of the energies it is.
b. Indicate which energies in Figure 13 are potential energies.
c. Use diagram 13 to determine Lars' maximum velocity and indicate the height at which he reaches this velocity.
d. Check if Jesse's diagram matches the data in Figure 12.

Suppose we don't leave the air friction out of the equation.

e. Explain which lines in figures 12 and 13 will change as a result.

FIGURE 13 THE RELATIONSHIP BETWEEN ENERGY AND HEIGHT FOR A JUMP WITH THE BUNGEE TRAMPOLINE

4. UNIVERSAL GRAVITATION

4.1 GRAVITATIONAL FORCE AND GRAVITATIONAL ENERGY

When you look at the movement of the moon around earth (Figure 14), you know that there must be a resultant force acting on the moon that causes it to make a circular motion. This resulting force is also called the **centripetal Force** F_c. Previously, you have learned the following relation between this force, velocity, mass and distance:

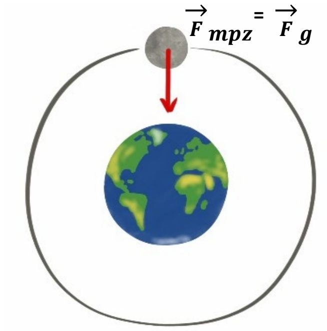

FIGURE 14 THE MOVEMENT OF THE MOON AROUND THE EARTH

$$F_c = \frac{mv^2}{r} \qquad (8)$$

F_c is the centrifugally searching force in N
m is the mass in kg
v is the velocity in m/s
r is the radius of curvature in m

This centripetal force is always provided by the sum of all forces that work on the object. If we look at the moon, there's only one force that plays a role here, that's the **gravitational force** F_g. The gravitational force is the general version of gravity from Chapter 2; it is the force due to mass exerted by an object on another object at a distance r . This gravitational force is defined as:

$$F_G = G\frac{m_1 m_2}{r^2} \qquad (9)$$

F_G is the gravitational force in N
G is the gravitational constant ($6{,}67384 \cdot 10^{-11}$ Nm^{2}kg^{-2})
m_1 and m_2 is the mass of object 1 or 2 in kg
v is the velocity in m/s
r is the distance between the center of gravity in m

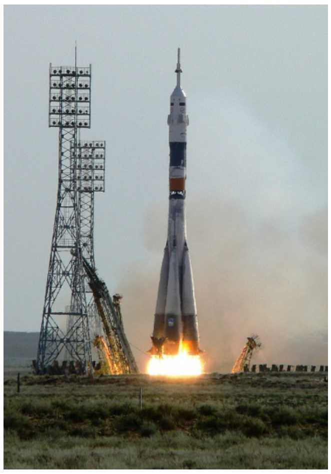

FIGURE 15 THE LAUNCH OF THE SOYUZ

By Yuri Chigrin, CC BY-SA 3.0.

You use this formula instead of formula (3), when dealing with objects at a great distance from the earth, or large changes in distance (e.g. when launching a rocket as in figure 15). In that situation the gravitational acceleration is not 9.81 m/s^2 and therefore you need to use the gravitational constant G and the distance to the center of gravity of the earth. This also applies to calculations with energy. The energy that an object has, at a certain distance from another object, is called the **gravitational energy** E_G. As with gravitational force, gravitational energy is a general version of Earths gravitational energy. Gravitational energy is defined as follows:

$$E_G = -G\frac{m_1 m_2}{r} \qquad (10)$$

EXCERCISE 11

Using formula (8) and (9), explain that the moon's orbital velocity does not depend on the mass of the moon.

EXCERCISE 12

Derive from formula (9) indicates that the fall acceleration (g) on the earth's surface is approximately equal to 9,81 m/s^2.

4.2 Potential energy

Gravitational energy is potential energy; it is a measure of the amount of work that can be performed on an object by the gravitational force. You have seen this before with Earths gravitational energy: If you hold an object at a certain height and then release it, the object will fall. The gravity that works on the object causes the object to accelerate. Gravity then performs work, converting the potential energy into velocity, i.e. kinetic energy. The same takes place for an object that is far from the Earth, although the formula is a bit more complicated, because the acceleration is not constant.

As with Earths gravitational energy, the zero point of gravitational energy can be chosen freely, because it is not about the absolute value, but a difference in energy. The zero point has been chosen to be at $r = \infty$, because the gravitational force is also zero at infinite distance from a mass. The value of the gravitational energy is therefore a measure of the amount of energy needed to escape the gravitational force of an object. This choice of zero point results in a negative gravitational energy.

Example: Launch of a satellite (without friction)

When we look at the launch of a geostationary satellite, chemical energy is converted into kinetic energy during this launch. Due to the kinetic energy, the satellite goes up and kinetic energy is converted into gravitational energy. This continues until the gravitational force on the satellite is exactly the same as the required centrifugal force, this is the case at an altitude of about 36000 km. From that moment on, the satellite continues to move around the Earth (Figure 16). Figure 17 shows the kinetic and gravitational energy of the satellite relative to the height of the satellite.

Figure 16 The orbit of a satellite launch

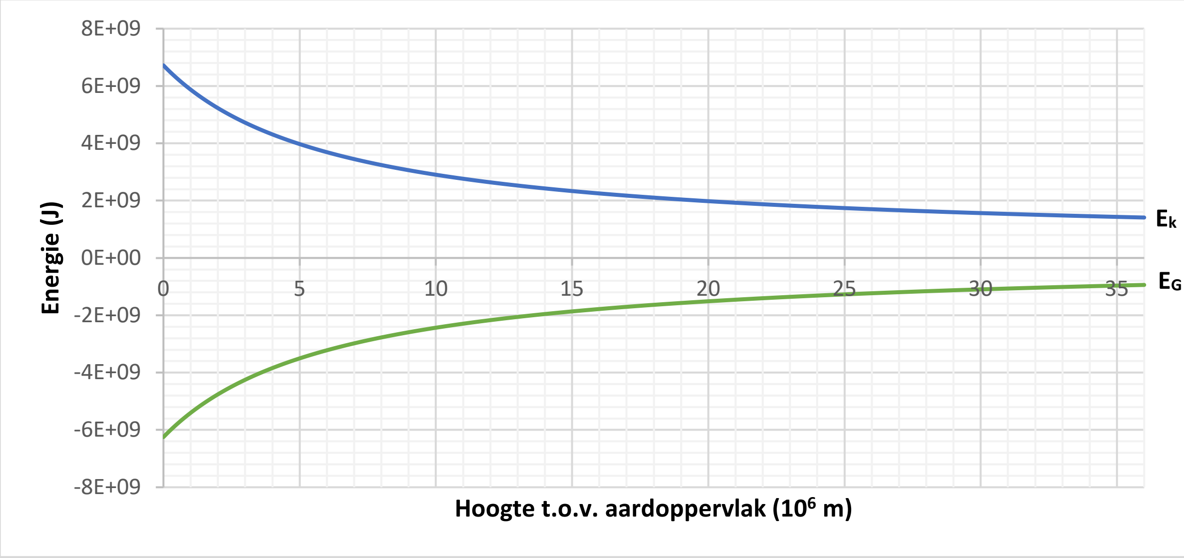

Figure 17 The energy of a satellite in the vicinity of the Earth

Exercise 13

Explain how you can see that figure 17 does not take into account air friction.

EXCERCISE 14

You have previously learned that a geostationary satellite is a satellite with a specific orbital time, velocity and altitude.

a. Explain what a geostationary satellite is and what orbital time is associated with it.
b. Calculate the exact height of the orbit of a geostationary satellite.
c. Calculate the velocity of a geostationary satellite.
d. Use your answer to calculate how much energy per kilogram it takes to put a satellite into geostationary orbit.

EXCERCISE 15

A geostationary satellite is brought to the proper distance from Earth in several steps. First, the satellite is put into low Earth orbit. In order to get the satellite to the right distance from Earth, an internal combustion engine is turned on several times. Due to the forces that this imposes on the satellite, the satellite eventually enters its final orbit (see Figure 18). In low orbit, the satellite is located on 2000 km above the earth's surface. In the final orbit, the satellite is located on 36000 km above the earth's surface. Figure 19 the energy per kg is given from an object that moves around the Earth in a circular motion.

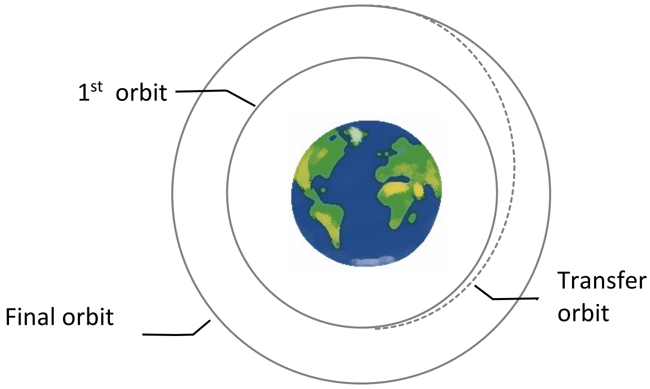

FIGURE 18 A SIMPLIFIED REPRESENTATION OF THE MOVEMENT OF A SATELLITE FROM A LOW TO GEOSTATIONARY ORBIT

a. Use Figure 19 to determine how much energy is required to move a rocket from orbit 1 to its final orbit.

Theoretically, when in its final orbit, the satellite would always continue to move in this orbit. In practice, an engine is needed to keep the satellite in its geostationary orbit.

b. Reason which factors may affect the orbit of the satellite.

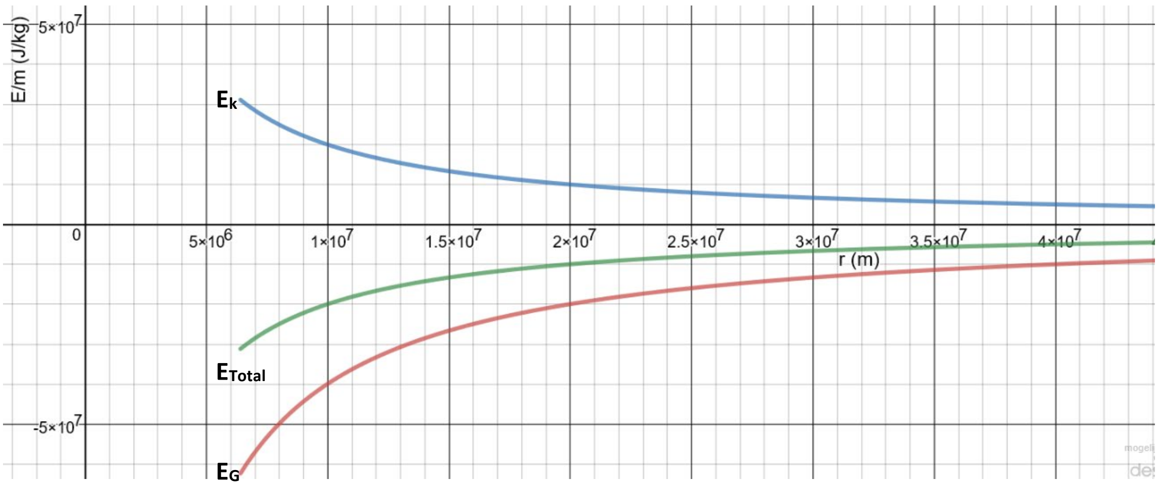

FIGURE 19 THE ENERGY PER KG OF AN OBJECT IN A CIRCULAR MOTION AROUND THE EARTH

4.4 SPACE PROBES

Space probes are unmanned spacecraft used to explore the universe. These space probes do not orbit earth, but are located near other planets or even outside our solar system. To launch these space probes, they must be launched at the right velocity; the escape velocity.

EXERCISE 16

a. Derives the escape velocity formula using formula (10) and a formula from BINAS.
b. In the diagram on the worksheet, sketch the kinetic and gravitational energy relative to the height of the space probe. Neglect the friction and clearly indicate what the differences are compared to Figure 17.

4.5 MANNED SPACE TRAVEL

Since 1961 there have been several manned space flights. Initially, these were flights in orbit around the Earth. The first manned flight to leave orbit was Apollo 8. During this flight, the spacecraft circled the moon. Since then, 12 astronauts have entered the lunar surface. So far, the moon has been the farthest destination for manned spaceflight, but by 2030 NASA plans to conduct a manned space mission to Mars. To do this, the space shuttle must completely leave earth's gravitational field.

EXERCISE 17

Figure 20 shows a diagram showing the potential energy of the Earth and the moon. Use this figure to explain whether it takes more, less or an equal amount of energy to land a space shuttle on the moon than to launch it into space.

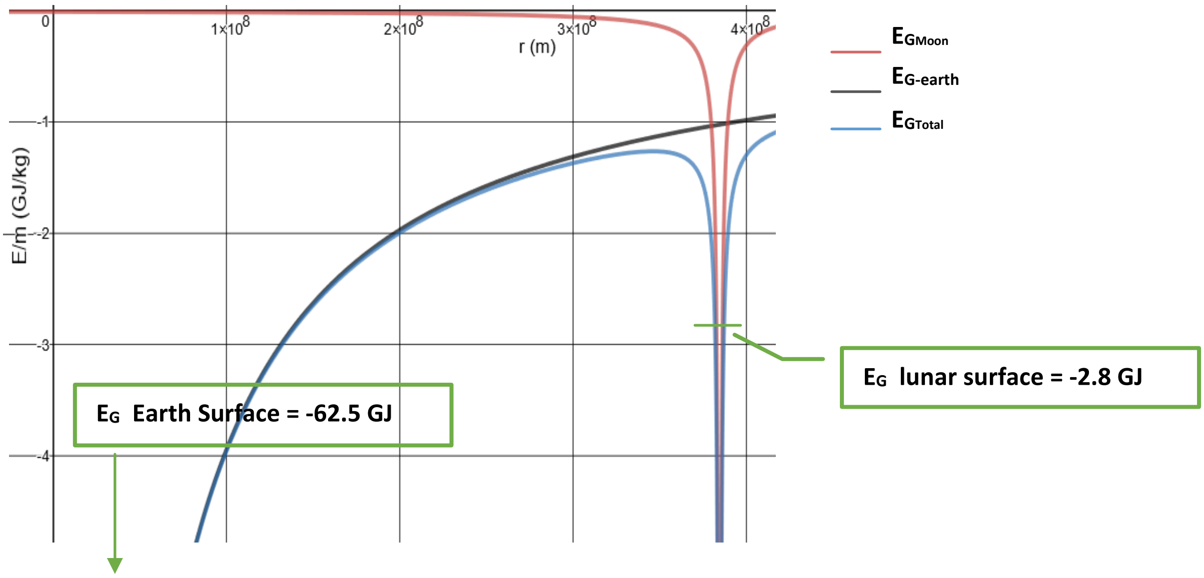

FIGURE 20 THE GRAVITATIONAL ENERGY PER KG OF AN OBJECT LOCATED IN THE VICINITY OF THE EARTH AND THE MOON

5. INTERMEZZO

5.1 FORCE AND POTENTIAL ENERGY

You may have noticed that the formulas of the force on an object and the potential energy in this module are very similar. Below is a brief overview of the formulas covered:

FORCE	**ENERGY**
$F_z = mg$	$E_z = mgh$
$F_v = -Cu$	$E_v = \frac{1}{2}Cu^2$
$F_G = G\frac{m_1 m_2}{r^2}$	$E_G = -G\frac{m_1 m_2}{r}$

There's a connection between force and energy. As explained earlier, E_p is the potential to perform work. A force acting on an object causes E_p to change. It follows that the force is the **derivative** of the E_p, i.e.:

$$F = -\frac{dE}{dx} \qquad (11)$$

This derivative is the **slope** of the energy. As a result, you can determine the force in an E,x-diagram by determining this slope. In reverse, you can determine the energy from an F,x-diagram by integrating or determining the **surface** below the graph.

EXAMPLE: ELASTIC FORCE AND ENERGY

When we look at elastic force and energy, we know the potential energy of a spring equals $E_v = \frac{1}{2}Cu^2$. If we determine the derivative to you, it is equal to:

$$-\frac{d}{du}\left(\frac{1}{2}Cu^2\right) = -2 \cdot \frac{1}{2}Cu^{2-1} = -Cu$$

Now, when we look at Figure 21, it can be seen that the surface area from 0 to 3 m is equal to $\frac{1}{2} \cdot 3 \cdot -1{,}5 = -2{,}25\ Nm$. This is equal to -1·$E_p$ at u = 3 m in figure 22.

If you look at the tangent in figure 22, its slope is equal to $\frac{-2{,}0}{2{,}0} = -1\ J/m$. This is equal to -1·F on *you* = -2 m in Figure 21.

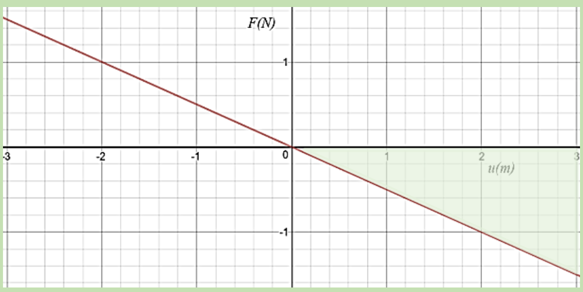

FIGURE 21 THE RELATIONSHIP BETWEEN THE FORCE AND DISPLACEMENT OF A SPRING

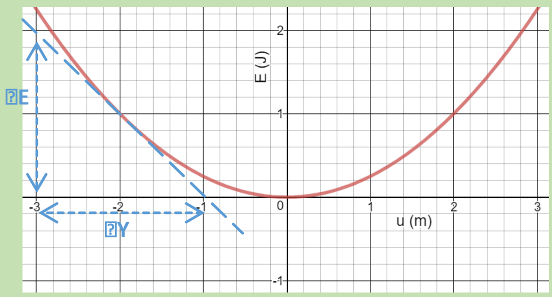

FIGURE 22 THE RELATIONSHIP BETWEEN THE POTENTIAL ENERGY AND THE DISPLACEMENT OF A SPRING

Note: You may notice that the minus sign is missing at gravity and gravitational force, this has purely to do with the choice to define a force towards the center of the earth as positive.

6. Electric Energy

6.1 Force and energy of two point charges

When two charged particles come close together, they exert an **electrical force** on each other. Previously, you have learned that this applies:

$$F_{el} = f\frac{q_1 q_2}{r^2} \quad (12)$$

F_{el} is the force that the charges exert on each other in N
f is the constant of Coulomb ($8{,}988 \cdot 10^9$ Nm^2C^{-2})
q_1 and q_2 is the load of particle 1 or 2 in C
r is the distance in m

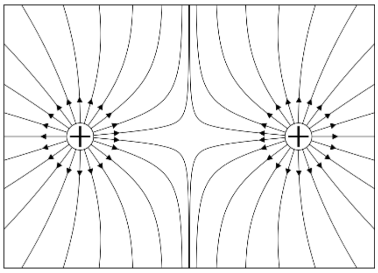

FIGURE 23 THE ELECTRIC FIELD OF SIMILAR POINT CHARGES

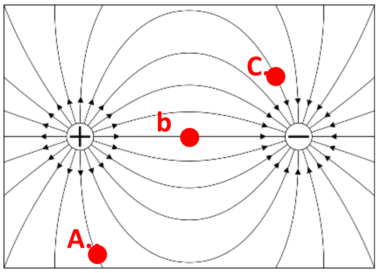

FIGURE 24 THE ELECTRIC FIELD OF DISPARATE POINT CHARGES

Here, F_{el} is negative if the charges attract each other and positive if the charges repel each other. This equation is very similar to equation (9) for the gravitational force, but the electric force can can also be negative. In Figure 23 and 24 you can see the electric field lines associated with these situations. Previously, you learned that an object, which is under the influence of forces, possesses potential energy. These charged particles also have a potential energy, this energy is **electrical energy** E_{el} can be described as:

$$E_{el} = f\frac{q_1 q_2}{r} \quad (13)$$

Exercise 18

Figures 23 and 24 show electrical field lines.

a. Explain what electric field lines represent.

Figure 24 shows three points, a, b and c.

b. Set points a, b and c in order of decreasing electric field strength.

Exercise 19

Calculate the size of the electrical force that a hydrogen nucleus exerts on the electron in that atom. Use the Bohr radius a_0 from BINAS table 7.

Exercise 20

In the diagram on the work sheet, sketch the electrical energy relative to the distance to the nucleus of an electron in a hydrogen nucleus.

Exercise 21

Suppose you bring another charged particle close to an electron in a hydrogen atom. Explain whether this would change the diagram you outlined in exercise 20.

6.2 FORCE AND ENERGY IN A HOMOGENEOUS FIELD

When an electrically charged particle is placed in a homogeneous electric field, a force is exerted on this particle (Figure 25). This force F_{el} is the constant between the plates and can be calculated with:

$$F_{el} = \frac{qU}{d} \qquad (14)$$

F_{el} is the electric field force in N
q is the load of the particle in C
U is the potential difference between the plates in V
d is the distance between the plates in m

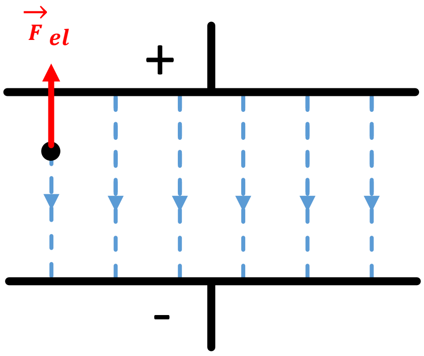

FIGURE 25 AN ELECTRON IN AN ELECTRIC FIELD

In this situation, an electric force acts on a particle (we leave gravity aside for now), so that the particle possesses potential energy. When the particle moves in between the plates, change in electrical energy is equal to:

$$\Delta E_{el} = qU \qquad (15)$$

EXERCISE 22

An electron is located in an electric field as shown in Figure 26 and moves from standstill from point A to point B. 12 V. The distance between the plates and positions A and B is true to scale. Gravity may be disregarded.

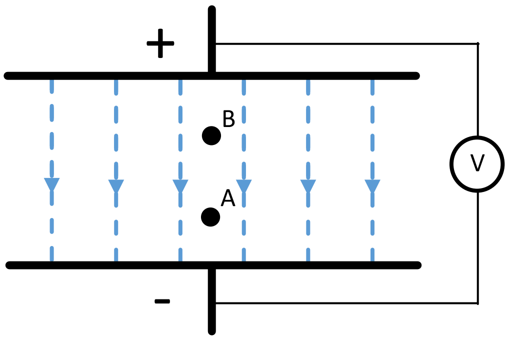

FIGURE 26 AN ELECTRON MOVES IN A HOMOGENEOUS ELECTRIC FIELD

a. Calculate the velocity that the electron has at position B.
b. In the diagram on the worksheet, sketch the kinetic and electrical energy related to the position between the plates. zero point of the electrical energy may be chosen at random.
c. Explain how, from the diagram you drew at exercise 22b., you can see that F_{el} is equal everywhere between the plates.

6.3 ALPHA DECAY

EXERCISE 23

Beryllium-8 (^8_4Be) is a highly unstable atom that emits alpha radiation. During the alpha decay, an alpha particle (^4_2He) escapes from the nucleus of an atom. Subsequently, the remaining core is equal to ^4_2He:

$$^8_4Be \rightarrow 2^4_2He$$

Just before the escape of the alpha particle, we can think of the core of Beryllium-8 as two alpha particles that are close to each other. Both have a load of $+2e$ and a mass of $6.6 \cdot 10^{-27}$ kg. The maximum distance between these particles is equal to the diameter of the core, which is approximately $4 \cdot 10^{-15}$m.

a. In the diagram on your worksheet, sketch the potential energy related to the distance between the two particles.

Figure 27 below shows an approximation of the actual potential energy of the alpha particle.

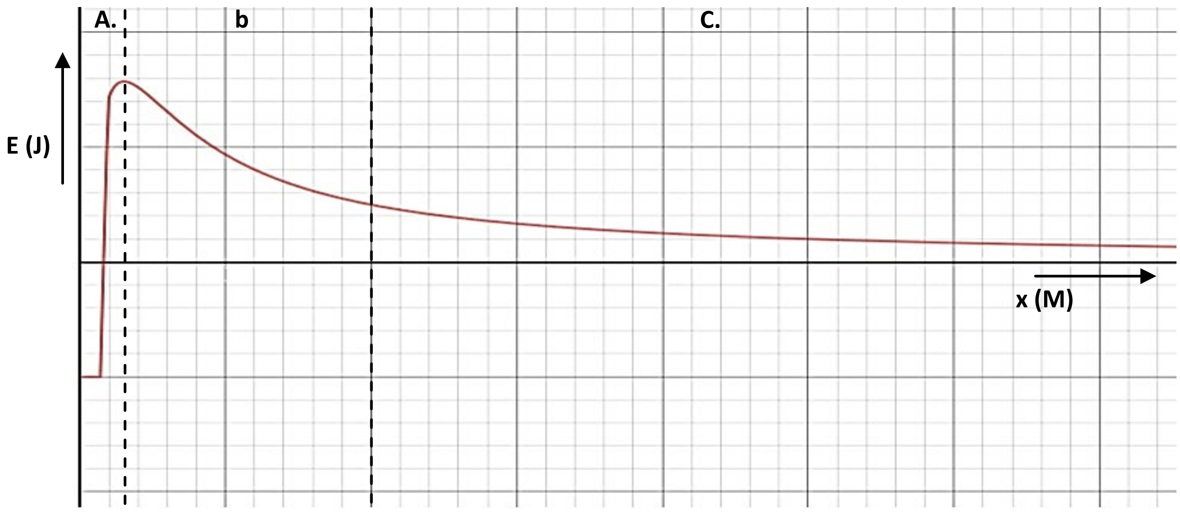

FIGURE 27 THE POTENTIAL ENERGY OF AN ALPHA PARTICLE IN THE NUCLEUS

b. Explain that this diagram shows that more forces play a role than just the electrical force.

Figure 26 shows three areas; areas A, B and C.

c. For each of these areas in the table on the worksheet, indicate whether the particle in that area is experiencing a repellent or attracting force, and explain why.

d. Rank the three areas in order of decreasing mean kinetic energy.