DZLM German Center for Mathematics Teacher Education

Leibniz Gemeinschaft

IPN Leibniz Institute for Science and Mathematics Education

Developing understanding for procedures through mathematical connections

Design research for mathematics classrooms and teacher professional development

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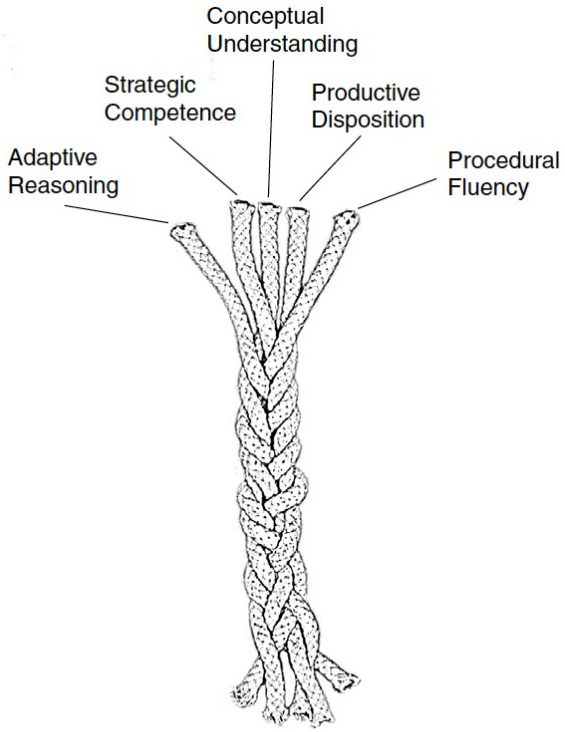
Math wars never end?

For low-performing students, we should ~~concentrate to~~ develop procedural fluency

And an important learning goal is the conceptual understanding

Also for procedures

No contradiction, we must connect both,
in particular for low-performing students



Intertwined Strands of Proficiency

(Kilpatrick, Swafford, Findel 2001)

Agenda for the talk

1. Two examples for introducing the idea of connecting procedures with understanding

2. Mastering Math - a design research project for fostering students' basic concepts in Grade 5-7

3. Mastering Math - a PD design research project for empowering teachers to foster students' basic concepts

4. Looking back on implementation strategies for bringing instructional innovations into classrooms

Introductory example: Multiply by 10

1. How do you speak about multiplying by 10 with your (struggling) students?

1329 x 10

To multiply by 10, simply add a zero in the end

To multiply by 10, push the decimal point one to the right

To multiply by 10, shift all place values one to the left

Question 1: www.menti.com with code 6616 5183
or QR Code:

Mentimeter

Introductory example: Multiply by 10

1. How do you speak about multiplying by 10 with your (struggling) students?
2. How would **your math teacher (students)** speak about it?

1329 x 10

To multiply by 10, simply add a zero in the end

To multiply by 10, push the decimal point one to the right

To multiply by 10, shift all place values one to the left

Question 1: www.menti.com with code 6616 5183 or QR Code:

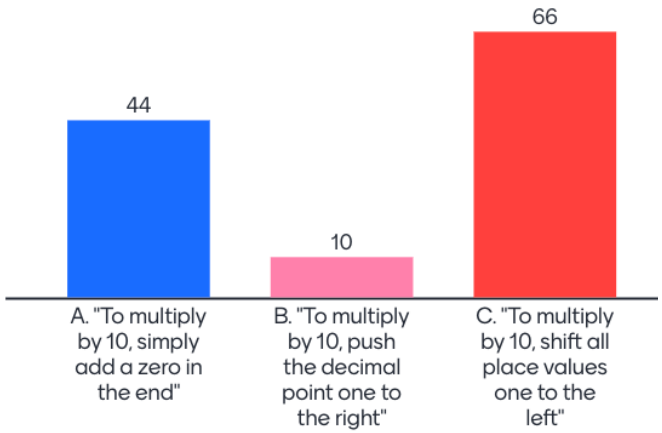
Mentimeter

Question 2: www.menti.com with code 6862 2439 or QR Code:

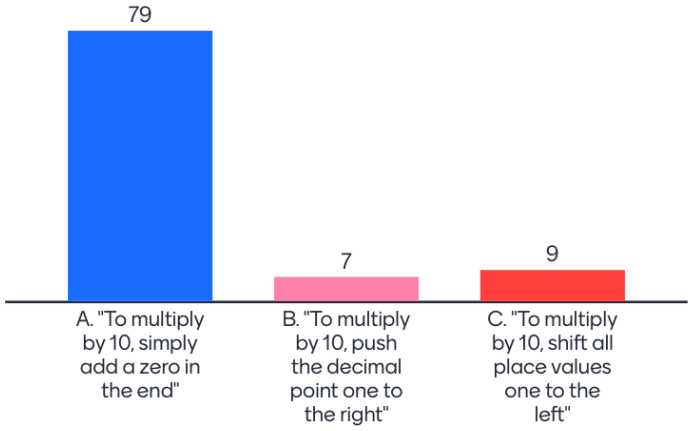
Mentimeter

Introductory example: Multiply by 10 – Answers in Zeist at Panama Conference

1. How do you speak about 1329 x 10 with your struggling students??



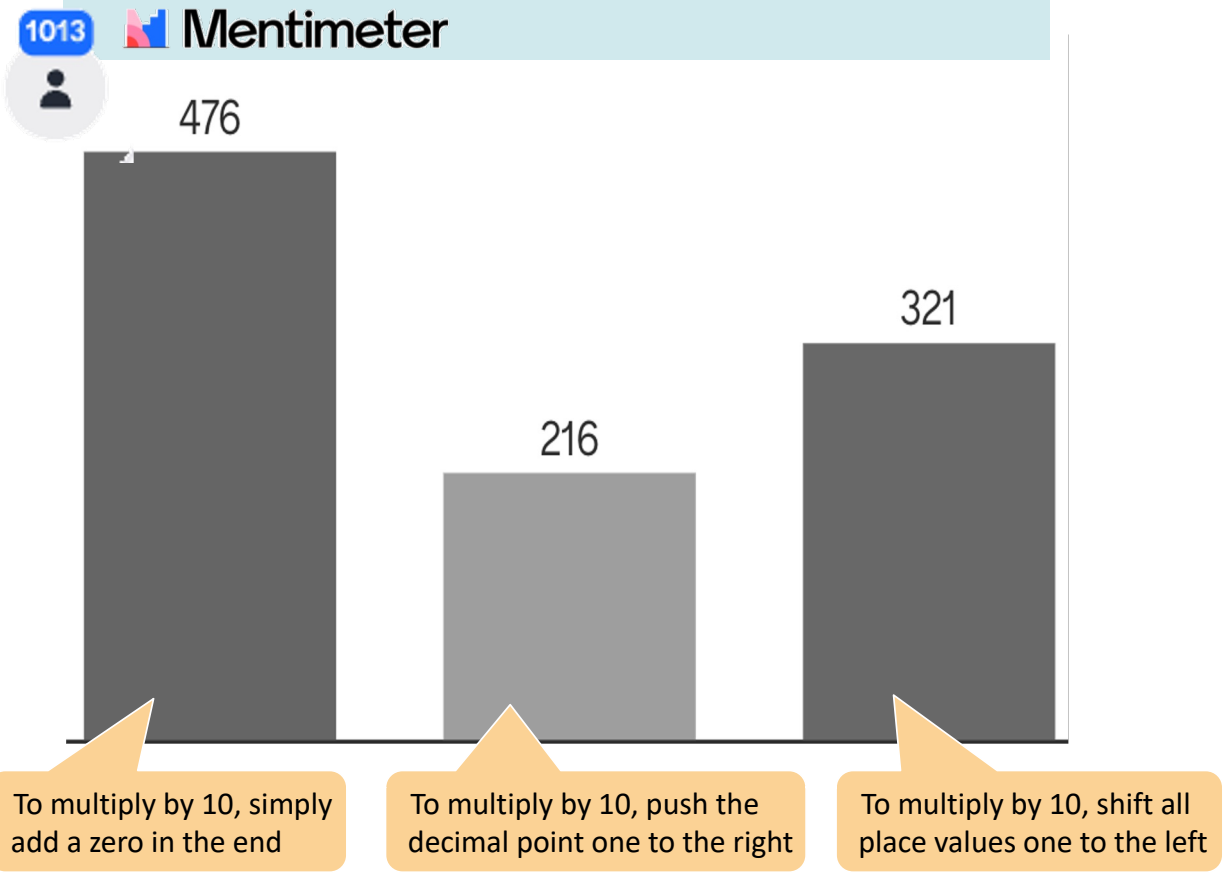
2. How would your math teacher (students) speak about 1329 x 10?



Introductory example: How do German teachers answer?

How do German mathematics teachers speak about multiplying by 10 with their (struggling) students?

2349 x 10



Longitudinal connections in the spiral curriculum

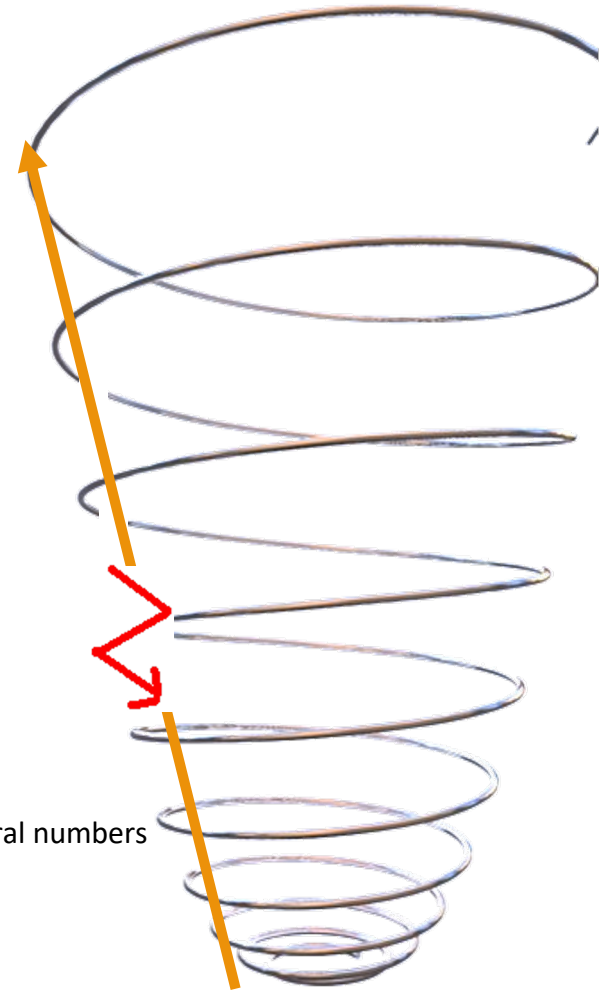
(Bruner 1966)

23.49×10

$= 23.490$

To multiply by 10, simply add a zero in the end

2349×10



Grade 6:
Decimal numbers

Grade 3-5: Natural numbers

Teachers‘ short-term and long-term oriented practices

2349 × 10

23.49 × 10?
= 23.490 ✗

To multiply by 10, simply add a zero in the end

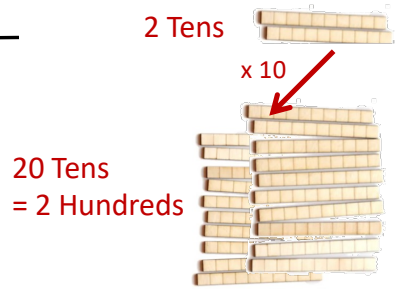
To multiply by 10, push the decimal point one to the right

Explain the rule with base-ten blocks

H	T	O	t	h
	2	3.	4	9
x 10	↓	↓	↓	↓
2	3	4.	9	

To multiply by 10, shift all place values one to the left

I can understand what happens to the 2 tens



Superficial teaching optimized for immediate task completion in short-term orientation

Simple procedure leads to quick solutions
but is not usable for decimal numbers

unconnected from underlying basic concepts

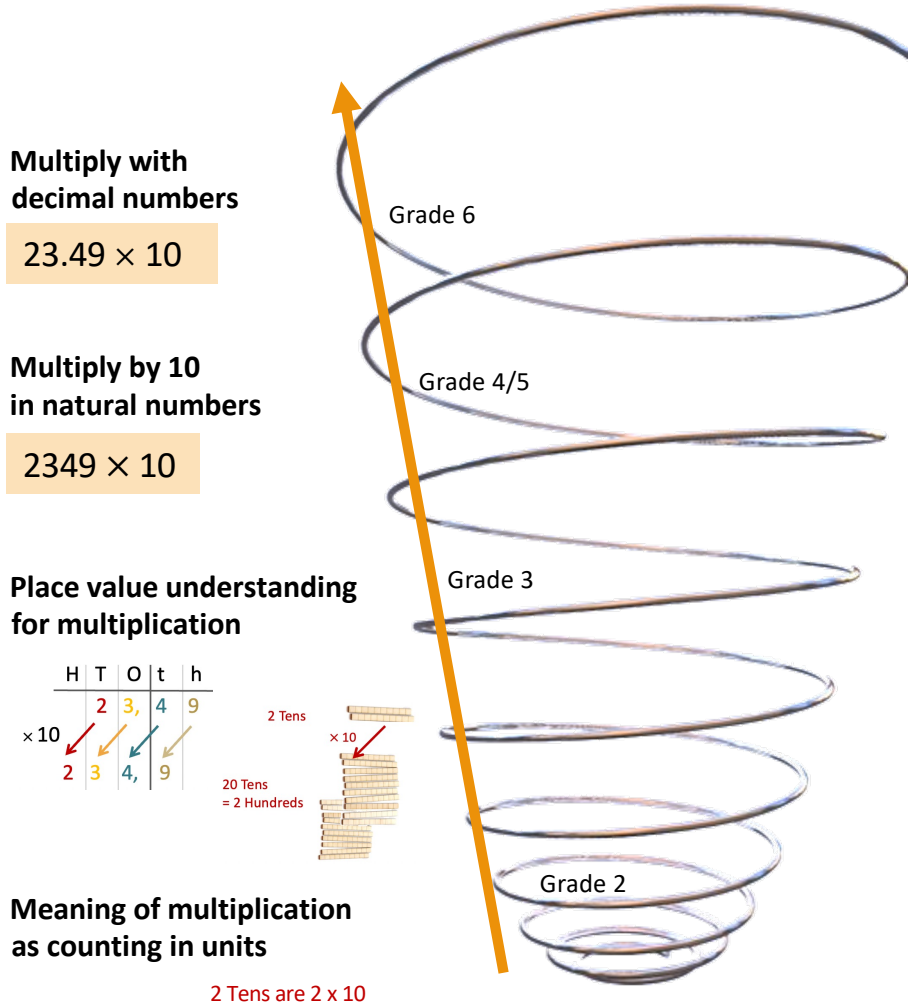
Deep learning optimized for progress along a learning trajectory in long-term orientation

Also a simple procedure,
but usable for natural and decimal numbers

and connected to basic concepts, here:
- place value understanding with base-ten unitizing
- meaning of multiplication as counting in units

Longitudinal connections in the spiral curriculum as a challenge for teachers

Understanding of basic concepts is crucial for long-term learning progressions



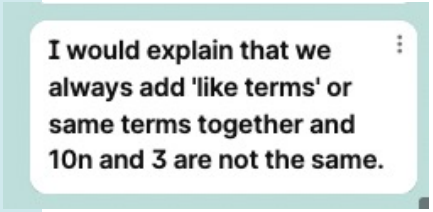
Second example: Algebraic transformation error

Activity for teachers:

This is a typical error of my former student Lea (15 year old):

$10n + 3 = 13n$ f

To what would **you** refer for working with Lea on her error in transforming expressions?



Yes, absolutely! But to what do you connect to explain why?

Second example: Algebraic transformation error

This is a typical error of my former student Lea (15 year old):

$10n + 3 = 13n$ f

Collection of answers in a German PD
To what would **you** refer for working with Lea on her error in transforming expressions?

Teachers‘ padlet entries

- Arbeit mit Symbolen
- versteckte Rechenzeichen einfärben
- Bei "Mal" ist es doch anders.
- gleichartige/ungleichartige Terme
- Gleiches zu Gleichem: Zahl zu Zahl; Variable zu Variable
- Unterschiedliche Einheiten
- Vergleich... Bananen zu Bananen und Äpfel zu Äpfel. z.B.
- Äpfel und Birnen
- Äpfel und Birnen

- Was bedeuten eigentlich die Variablen, warum gehören 10n und 3 nicht zusammen?
- **Was bedeutet n?**
- **was bedeutet 10n?**
- n+n+n+n+n+n+n+n+n+n+3= ....
- **Die Frage ist wozu sie es im Leben/Alltag wirklich übertragen können/sollen**
- **mangelnde Grundvorstellungen einer Variablen. begreifen Variable nicht als Platzhalter.**
- **Am Verständnis von Termen. Oft verwirren die Buchstaben, sie werden nicht als Platzhalter verstanden**
- Platzhalter durch etwas Reales ersetzen
- **Terme so veranschaulichen, dass der Unterschied zwischen Variable und Zahlenwert klar wird. Mit Anschauung arbeiten**

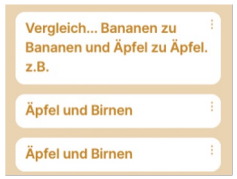
  Verstehenslücken aufarbeiten im Alltag ist auch wegen dem breiten Spektrum der Schüler oft schwierig. Vier Niveaus in einer Klasse bis zum Lernbehinderten

(Padlet entry of teachers participating in a PD in May 2022 of S. Prediger)

Work on making expressions understandable. Often, the letters are confusing, as not yet understood as place holders

Teachers' short-term and long-term oriented practices

Superficial learning optimized for immediate task completion in short-term orientation



Why not apple and bananas?

10 + 3 = 13 ???

Well-known source of typical misconceptions in the fruit salat algebra:

For each apple, we have 3 bananas 1 = 3

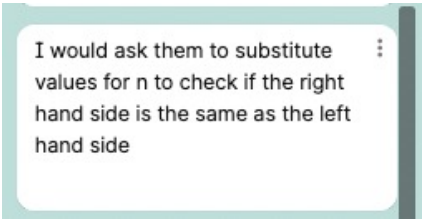
1 A = 3 B Or

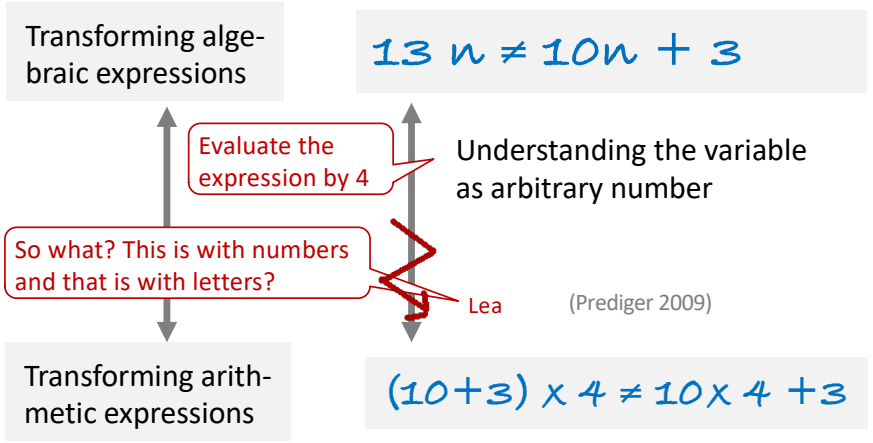
3 A = 1 B ?????

Unfaithful analogies not reflecting the mathematical structures destroy more than they help

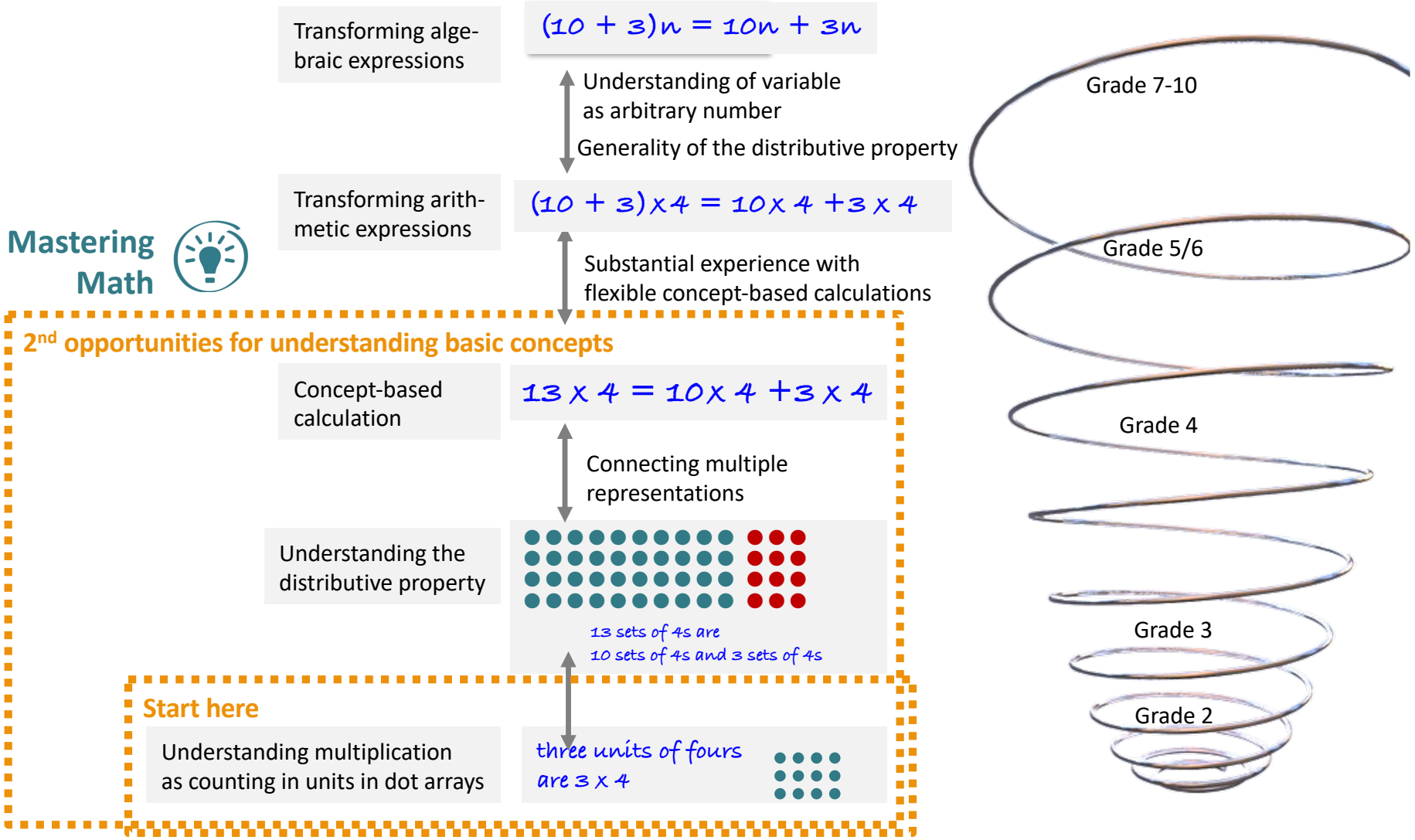
(Resnick & Clement, 1981; Chick 2009)

Deep learning optimized for progress along a learning trajectory in long-term orientation





Developing sustainable understanding requires basic concepts



Longitudinal connections in the spiral curriculum as a challenge for teachers

General phenomenon well-known,e.g., in South Africa:
“requirements of the high school curriculum make
it virtually impossible for learners who have been
disadvantaged by their early schooling to ‘catch-up’ later
sufficiently“
(Taylor, Muller & Vinjevold, 2003, p. 129)

I do not have the time to thoroughly work on these basic concepts

Mastering Math

Start here

Just, because time is always short,
let us not waste time and invest time wisely!

Understanding of basic concepts
is crucial for long-term learning progress

Multiply with decimal numbers

$23{,}49 \times 10$

Multiply by 10 in natural numbers

2349×10

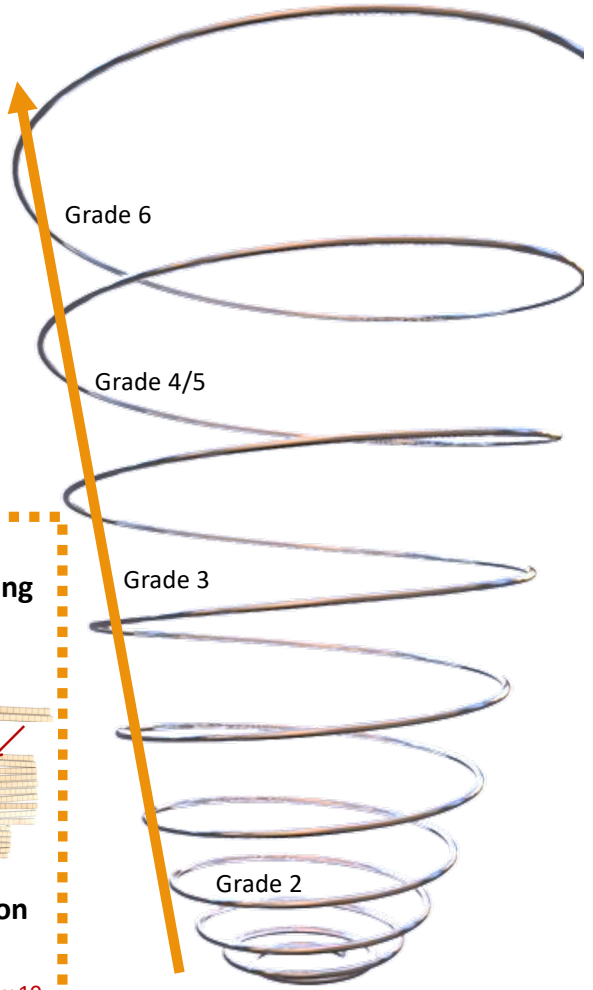
Place value understanding

	H	T	O	t	h
		2	3,	4	9
× 10					
	2	3	4,	9	

2 Tens
× 10
20 Tens
= 2 Hundreds

Meaning of multiplication as counting in units

2 Tens are 2 x 10



What do we learn from the two examples?

And others have found, too

van den Heuvel Panhuizen 2003
Kilpatrick et al. 2001
Spaull 2013 "vertically demarcated"

Kilpatrick et al. 2001
Baroody, Fell, Johnson 2007
Glade & Prediger 2017

Hiebert & Carpenter 1992

Askew 2019
Prediger & Buró 2021

Moser Opitz 2007,
Maccini et al. 2007
Taylor, Muller, Vinjevold 2003

Slavin & Madden 1989
Moser Opitz et al. 2017
Prediger et al. 2019

Boaler 2002
Wilhelm et al. 2007

Herbst 2003
Watson & de Geest 2005
Prediger & Buró 2021

Karsenty 2010
Prediger et al. 2022

Mathematics

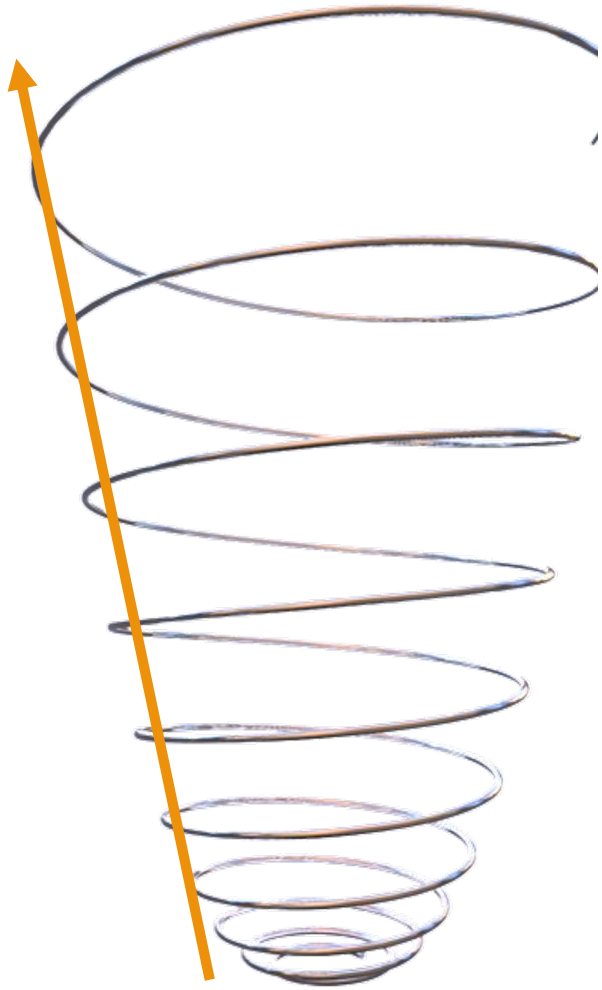
- is highly cumulative as higher learning contents are grounded on basic concepts in longitudinal learning trajectories
- in particular, procedures in higher grades need to be justified and grounded in basic concepts of earlier grades

Mathematics learning of at-risk students

- learning with understanding means connecting to already known ideas
- is necessarily reduced to superficial procedural learning if the basic concepts are not sufficiently acquired
- at-risk students often had too limited learning opportunities for basic concepts in earlier grades
- need second chances for learning basic concepts in later grades

Mathematics teaching of many teachers

- in particular for at-risk students, teaching practices are often mainly procedural with too low cognitive demands
- often shaped by superficial teaching practices optimized for immediate task completion in short-term orientation
- **but**: teachers' practices and orientations **can** be promoted and supported for developing long-term oriented practices



Agenda for the talk

1. Two examples for introducing the idea of connecting procedures with understanding
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Big questions for design researchers

Overall aim of the Mastering Math project (ongoing since 2010)

Provide productive learning opportunities for students and teachers
for basic arithmetic concepts to be understood in Grade 5-7 (some of them from Grade 2-4)

Classroom level

Design Research questions on the classroom level

- What do students have to learn?
- How can they learn it in an intervention,
  and what hinders them to learn it?
- Is the instruction really effective for students' learning gains?

Curriculum Material
in 45 modules in 20 topics

Natural numbers

N1	Place value understanding with base-ten blocks and place value table
N2	Place value understanding on the number line
N3	Understanding addition and subtraction in multiple representations and self-written word problems
N4	Understanding multiplication and division in multiple representations and self-written word problems
N5/6	Concept-based calculation strategies based on number sense
N7/8	Written algorithms

Fractions and decimal numbers

B1	Understanding fractions in part-whole-relationship (simple fractions and shares of sets)
B2	Understanding equivalence of fractions
B3	Understanding order of fractions
B4	Calculation with fractions
D1	Place value understanding for decimal numbers
D2	Understanding order of decimal numbers
D3	Understanding addition and subtraction of decimal numbers
D4	Understanding multiplication and division of decimal numbers
DB	Understanding the connection of decimal numbers and fractions

Word problems measurement percentages

S3	Understanding and calculating percentages
S5	Cracking word problems with comprehension strategies

...

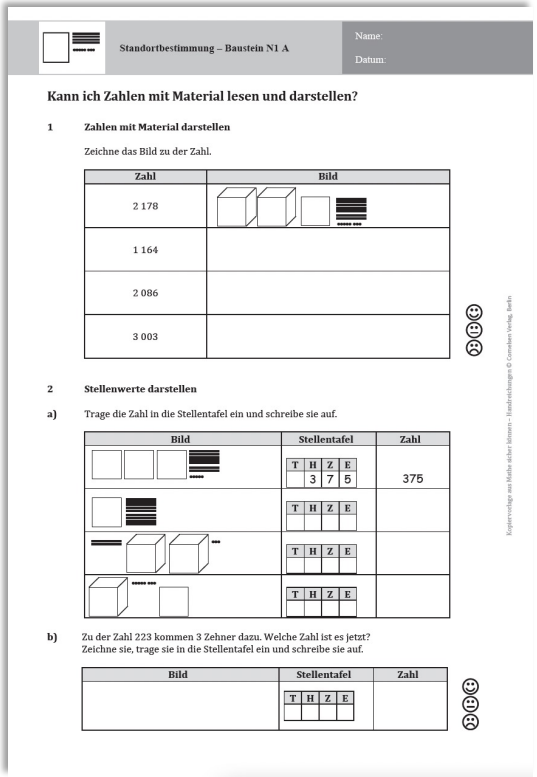
Selter, C., Prediger, S., Nührenbörger, M., & Hußmann, S. (Eds.). (2014). Mathe sicher können – Natürliche Zahlen. Förderbausteine und Handreichungen für ein Diagnose- und Förderkonzept zur Sicherung mathematischer Basiskompetenzen. Cornelsen. Open Educational Ressource. http://mathe-sicher-koennen.dzlm.de/002.

Curriculum Material in 45 modules with three parts, each

124

N1 Place value understanding

Formative Assessment (10 min, 2-3 diagnostic tasks)



Standortbestimmung – Baustein N1 A

Name:

Datum:

Kann ich Zahlen mit Material lesen und darstellen?

1 Zahlen mit Material darstellen

Zeichne das Bild zu der Zahl.

Zahl	Bild
2 178	
1 164	
2 086	
3 003	

2 Stellenwerte darstellen

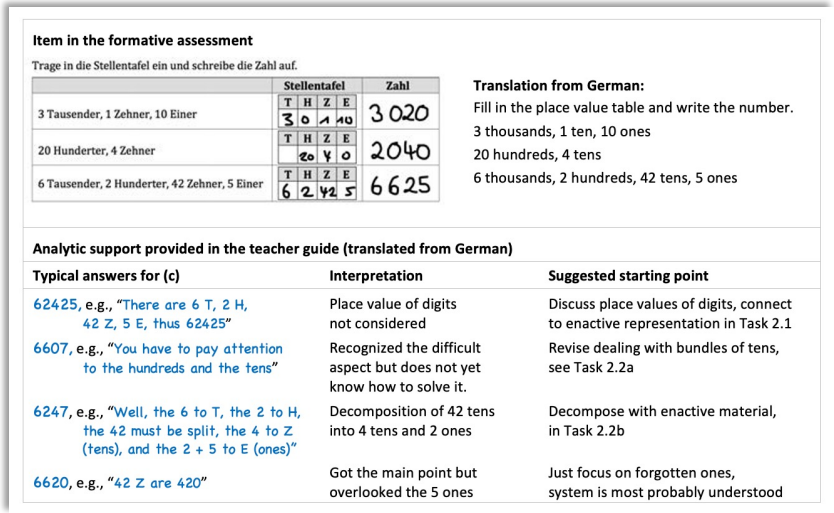
a) Trage die Zahl in die Stellentafel ein und schreibe sie auf.

Bild	Stellentafel	Zahl
	T H Z E: 3 7 5	375
	T H Z E	
	T H Z E	
	T H Z E	

b) Zu der Zahl 223 kommen 3 Zehner dazu. Welche Zahl ist es jetzt? Zeichne sie, trage sie in die Stellentafel ein und schreibe sie auf.

Bild	Stellentafel	Zahl
	T H Z E	

Teacher guide for interpretation (including backgrounds)



Item in the formative assessment

Trage in die Stellentafel ein und schreibe die Zahl auf.

	Stellentafel	Zahl
3 Tausender, 1 Zehner, 10 Einer	T H Z E: 3 0 1 10	3 020
20 Hunderter, 4 Zehner	T H Z E: 20 4 0	2040
6 Tausender, 2 Hunderter, 42 Zehner, 5 Einer	T H Z E: 6 2 42 5	6625

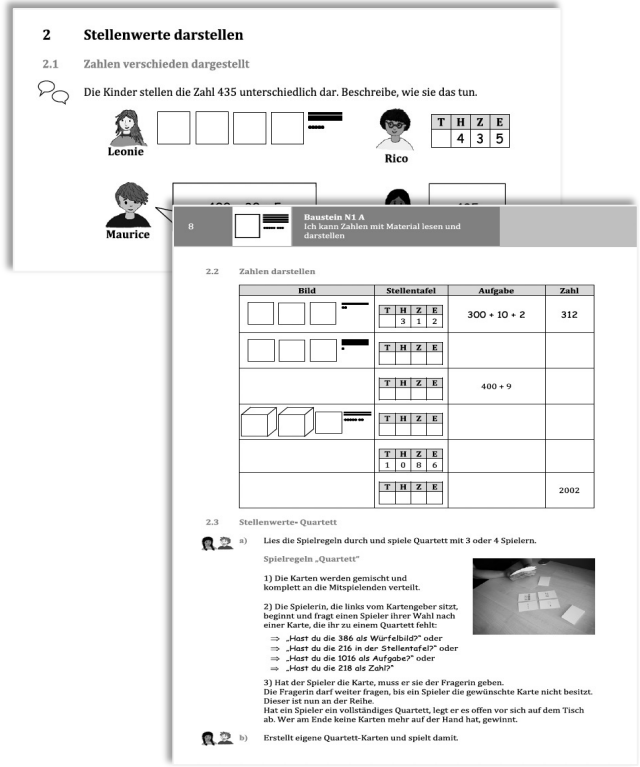
Translation from German:

Fill in the place value table and write the number.
3 thousands, 1 ten, 10 ones
20 hundreds, 4 tens
6 thousands, 2 hundreds, 42 tens, 5 ones

Analytic support provided in the teacher guide (translated from German)

Typical answers for (c)	Interpretation	Suggested starting point
62425, e.g., "There are 6 T, 2 H, 42 Z, 5 E, thus 62425"	Place value of digits not considered	Discuss place values of digits, connect to enactive representation in Task 2.1
6607, e.g., "You have to pay attention to the hundreds and the tens"	Recognized the difficult aspect but does not yet know how to solve it.	Revise dealing with bundles of tens, see Task 2.2a
6247, e.g., "Well, the 6 to T, the 2 to H, the 42 must be split, the 4 to Z (tens), and the 2 + 5 to E (ones)"	Decomposition of 42 tens into 4 tens and 2 ones	Decompose with enactive material, in Task 2.2b
6620, e.g., "42 Z are 420"	Got the main point but overlooked the 5 ones	Just focus on forgotten ones, system is most probably understood

Teaching material (2-3 pages for every diagnostic task)



2 Stellenwerte darstellen

2.1 Zahlen verschieden dargestellt

Die Kinder stellen die Zahl 435 unterschiedlich dar. Beschreibe, wie sie das tun.

Leonie

Rico

T	H	Z	E
	4	3	5

Maurice

8 Baustein N1 A – Ich kann Zahlen mit Material lesen und darstellen

2.2 Zahlen darstellen

Bild	Stellentafel	Aufgabe	Zahl
	T H Z E: 3 1 2	300 + 10 + 2	312
	T H Z E		
	T H Z E	400 + 9	
	T H Z E		
	T H Z E: 1 0 8 6		
	T H Z E		2002

2.3 Stellenwerte-Quartett

a) Lies die Spielregeln durch und spiele Quartett mit 3 oder 4 Spielern.

Spielregeln „Quartett"

1) Die Karten werden gemischt und komplett an die Mitspielenden verteilt.

2) Die Spielerin, die links vom Kartengeber sitzt, beginnt und fragt einen Spieler ihrer Wahl nach einer Karte, die ihr zu einem Quartett fehlt:
⇒ „Hast du die 386 als Würfelbild?" oder
⇒ „Hast du die 216 in der Stellentafel?" oder
⇒ „Hast du die 1016 als Aufgabe?" oder
⇒ „Hast du die 218 als Zahl?"

3) Hat der Spieler die Karte, muss er sie der Fragerin geben. Die Fragerin darf weiter fragen, bis ein Spieler die gewünschte Karte nicht besitzt. Dieser ist nun an der Reihe.
Hat ein Spieler ein vollständiges Quartett, legt er es offen vor sich auf dem Tisch ab. Wer am Ende keine Karten mehr auf der Hand hat, gewinnt.

b) Erstellt eigene Quartett-Karten und spielt damit.

Methodology: Topic-specific Design Research

Dual Aims

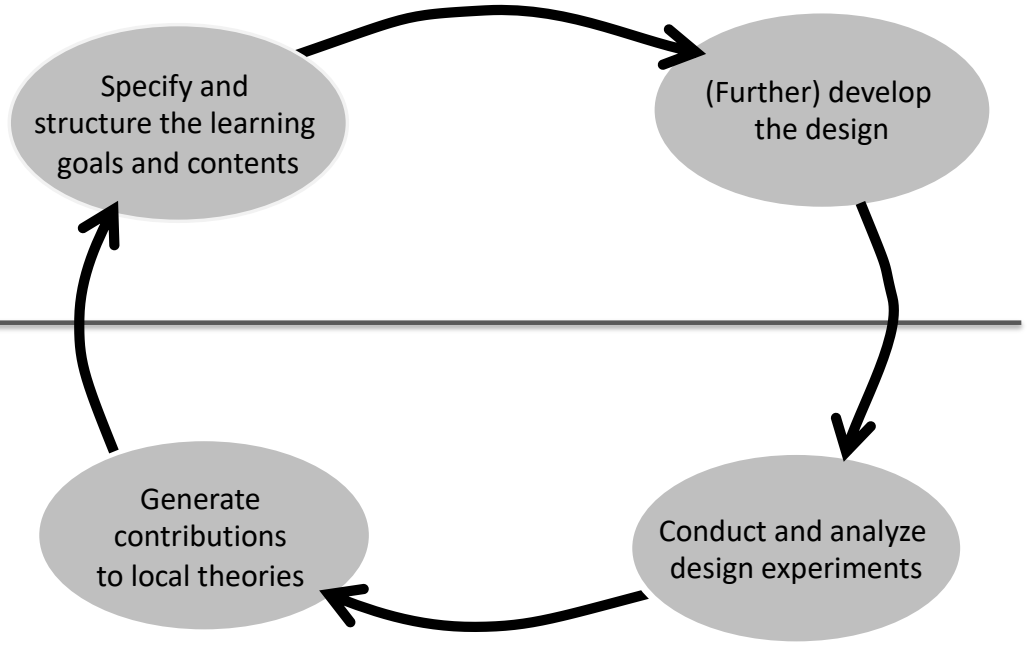
Research-based design

Develop and optimize learning environments, in which at-risk students in Grade 5-7 can develop understanding for basic arithmetics

Empirical insights into typical learning pathways and obstacles as well as functioning of the design elements

Design-based research

Working Areas of topic-specific Design Research



Prediger, S., & Zwetzschler, L. (2013). Topic-specific design research with a focus on learning processes: The case of understanding algebraic equivalence in grade 8. In T. Plomp & N. Nieveen (Eds.), *Educational Design Research: Illustrative Cases* (pp. 407-424). SLO, Netherlands Institute for Curriculum Development.
Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes – an overview on achievements and challenges. *ZDM – Mathematics Education*, *47*(6), 877–891 https://doi.org/10.1007/s11858-015-0722-3

Design Research on the classroom level for Mastering Math

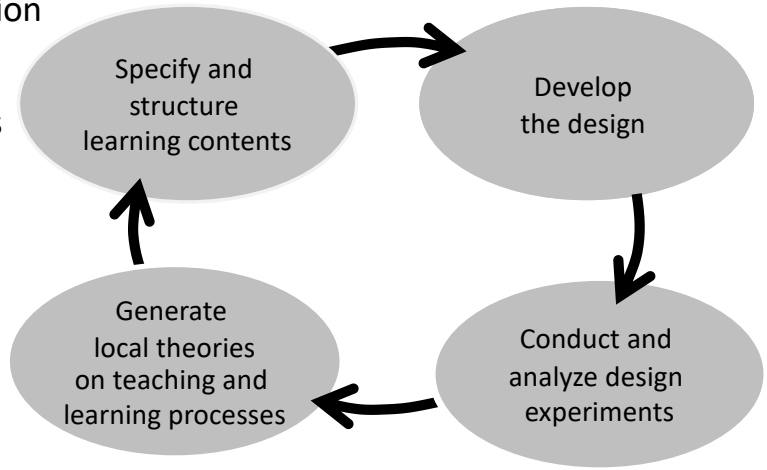
Conceptual focus

Students should learn to…

… understand basic concepts such as the place value system or meaning of multiplication & division

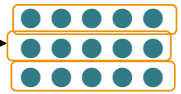
… explain & justify procedures by connecting them to basic concepts

Design principles	Realization in design elements
Conceptual focus	Connect multiple representations
Student focus	~~Open inquiry-based learning~~
	Deep learning building upon students' ideas

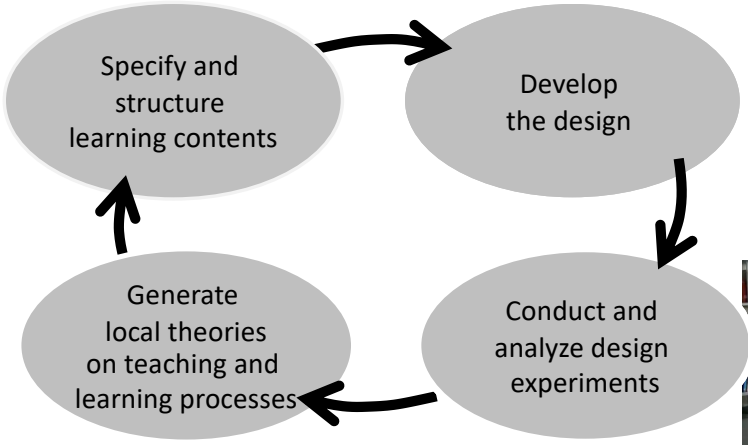


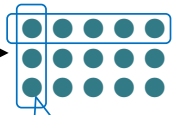
Design Research Cycles on Meaning of Multiplication

Understand meaning of multiplication
in ~~dot arrays~~
as counting in units in dot arrays

3 x 5 ⟷ 

Conceptual focus	Connect multiple representations
Student focus	Deep learning building upon students' ideas
Enhance communication	Engage students in rich discourse practices in teacher-led small groups



3 x 5 ⟷ 

Why does it fit?

Here is 3 and here is 5

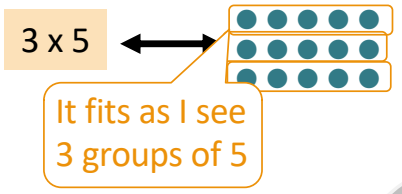
Connecting representations and explaining meanings requires to make the mathematical structure explicit (here, multiplicative unit)

but also, e.g..

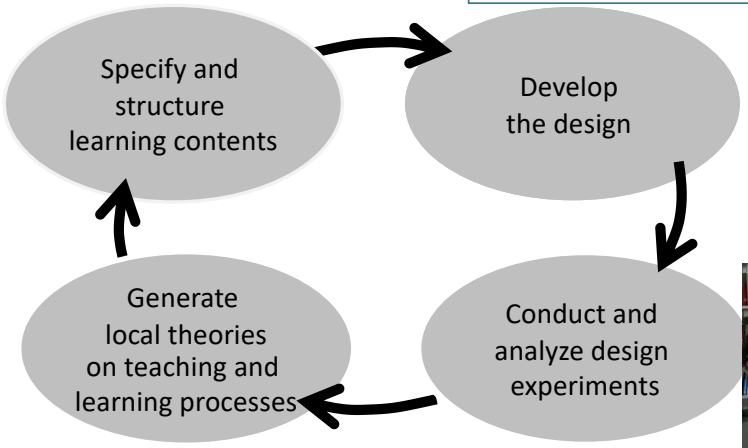
- part-whole structure for fractions (Prediger 2022), percentages (Pöhler & Prediger 2015) and conditional probability (Post & Prediger 2022)
- structure of functional relationships (Prediger & Zindel 2017) and diffferent of amount and change (Prediger & Şahin-Gür 2020)
- structure of variables as unknowns and generalizers (Prediger & Krägeloh 2015)

Design Research Cycles on Meaning of Multiplication

Understand meaning of multiplication
as counting in units in dot arrays



Topic-specific academic language in meaning-related phrases



Conceptual focus

Student focus

Enhance communication

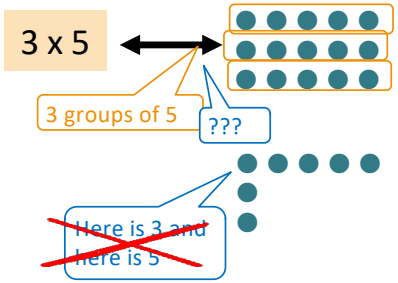
Challenge: How to support teachers to avoid superficial learning?

Connect multiple representations with a focus on structures

Allow deep insights into student thinking by rich diagnostic tasks

Engage students in rich discourse practices

and provide language support for their articulation by meaning-related phrases



Connecting representations requires to make mathematical structures

Articulating structures is hard for some students as they do not have the meaning-related phrases

(Prediger 2022 CERME plenary)

Design Research Cycles on Meaning of Multiplication

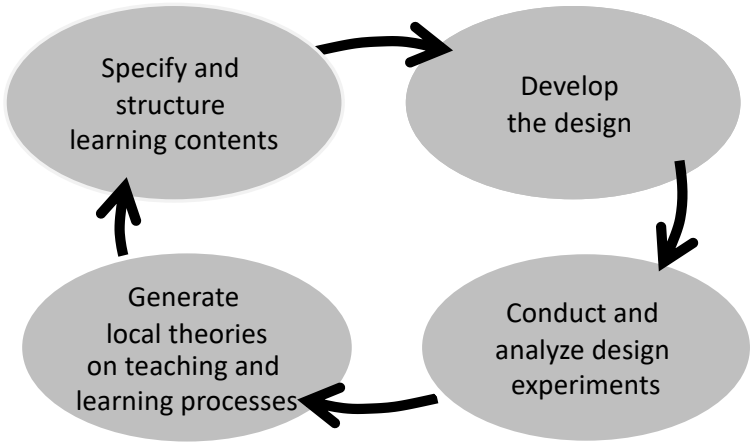
Conceptual focus

Students should learn to…

… understand of basic concepts such as place value understanding meaning of multiplication & division, …

… explain & justify procedures by connecting them to basic concepts

… use meaning-related language needed to explain & justify



Conceptual focus

Student focus

Enhance communication

Connect multiple representations with a focus on structures

Allow deep insights into student thinking by rich diagnostic tasks

Engage students in rich discourse practices and provide language support for their articulation by meaning-related phrases

Connecting requires to make the mathematical structures explicit

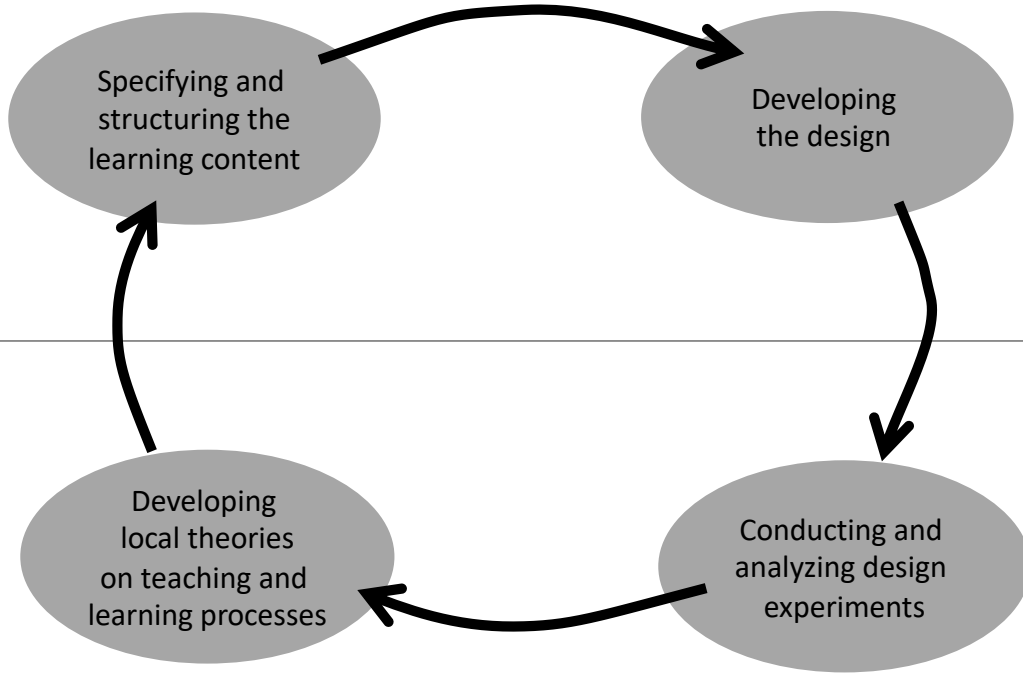
Articulating structures is hard for some students as they do not have the meaning-related phrases

(Prediger 2022CERME Plenary for Fractions, Prediger 2019 in Lernen & Lernstörung for multiplication)

Practical outcomes of Design Research

Design Process — 21 years of work, 7x 3 years → **Design Outcomes**

What-Questions? / How-Questions?

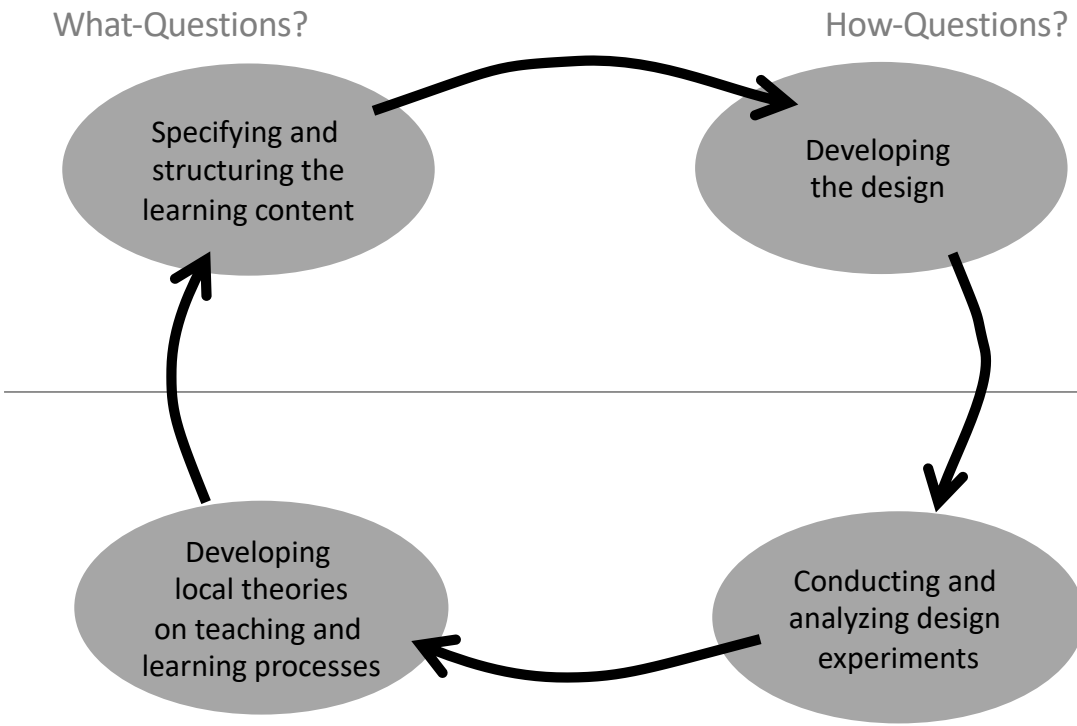


Specified and structured learning content
(here, for understanding basic concepts and underlying language requirements)

Teaching-learning arrangement
(here, 45 modules for concept-focused interventions)

Typical dual outcomes of Design Research

Research-based Design Process



Design-based Research Process

Design Outcomes

Specified and structured learning content
(here, for understanding basic concepts and underlying language requirements)

Teaching-learning arrangement
(here, 45 modules for concept-focused interventions)

Refined design principles
(applicable also for other contents)

Theory elements on typical topic-specific learning pathways and obstacles
(here, on the role of meaning-related language as epistemic catalyst)

Theory elements on topic-specific teaching processes
(with typical effects and conditions)

Research Outcomes

Prediger, S. (2019). Theorizing in Design Research: Methodological reflections on developing and connecting theory elements for language-responsive mathematics classrooms. *Avances de Investigación en Educación Matemática*, *8*(15), 5-27. https://doi.org/doi.org/10.35763/aiem.v0i15.265

Empirical evidence for effectiveness in a quasi-experimental field trial (2016)

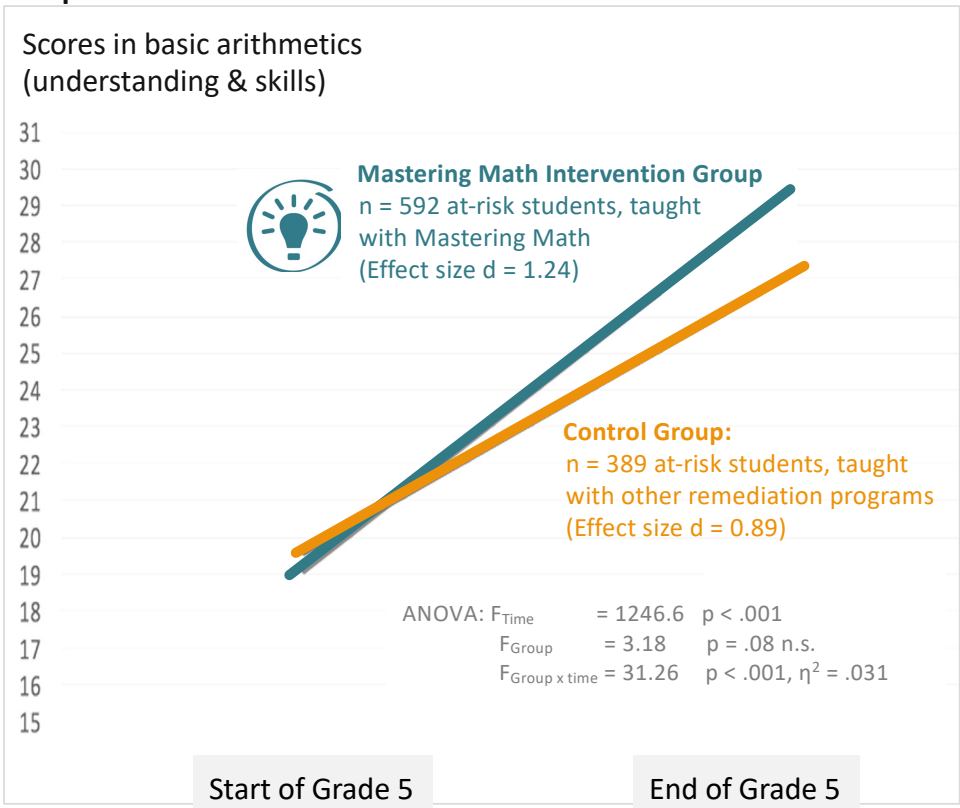
Research Question: Can we implement the **Mastering Math intervention** in real classrooms with regular teachers?

Initial sample
61 schools, ~ 120 mathematics teachers
N = 5785 fifth graders (all achievement ranges)

Sample of the field trial
881 at-risk students selected by low pretest score and fostered in small group sessions (~ 30 times, 40 min each)

Dependent variable

Scores in basic arithmetics (understanding & skills)



Results — Classroom level

Yes, we can!
Mastering Math Intervention Group
has significantly higher learning gains in understanding of basic concept and skills than **Control Group**

But: substantial differences between teachers
→ further PD research needed to improve targetedness of teachers' PD program

Teacher PD level

(Prediger, Fischer, Selter, & Schöber, 2019, in Educational Studies in Mathematics, 102(3), 361-378).

Agenda for the talk

1. Two examples for introducing the idea of connecting procedures with understanding

2. Mastering Math - a design research project for fostering students' basic concepts in Grade 5-7

3. Mastering Math - a PD design research project for empowering teachers to foster students' basic concepts

4. Looking back on implementation strategies for bringing instructional innovations into classrooms

DZLM

Big questions for design researchers

Overall aim of the Mastering Math project (ongoing since 2010)

Provide productive learning opportunities for students and teachers
for basic arithmetic concepts to be understood in Grade 5-7 (some of them from Grade 2-4)

Classroom level

Design Research questions on the classroom level

- What do students have to learn?
- How can they learn it in an intervention,
  and what hinders them to learn it?
- Is the instruction really effective for students' learning gains?

Teacher PD level

Design Research questions on the teacher PD level

- What do teachers have to learn?
- How can they learn it in PD programs
  and what hinders them to learn it?
- Is the learning really effective for teachers' professional growth?

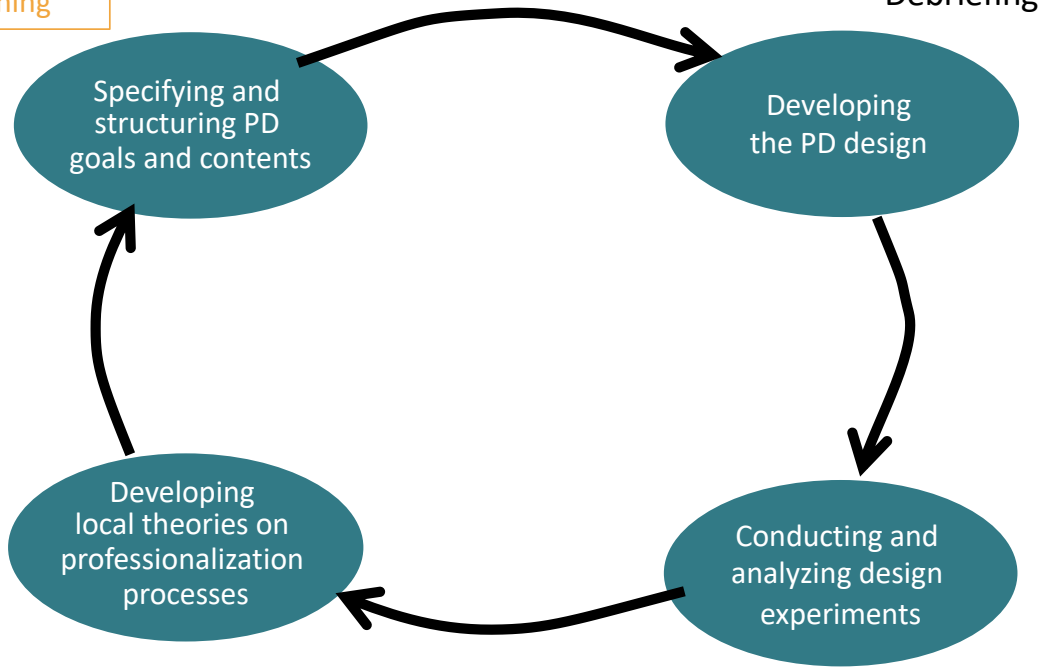
Working areas for content-related Design Research on the PD level

Design

Monitor students' learning

Enhance students' learning

Preparatory PD activities
Teacher's experiments with Mastering Math
Debriefing after experiments



Research

Prediger, S. (2019). Investigating and promoting teachers' pathways towards expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal, 31*(4), 367–392. doi:10.1007/s13394-019-00258-1

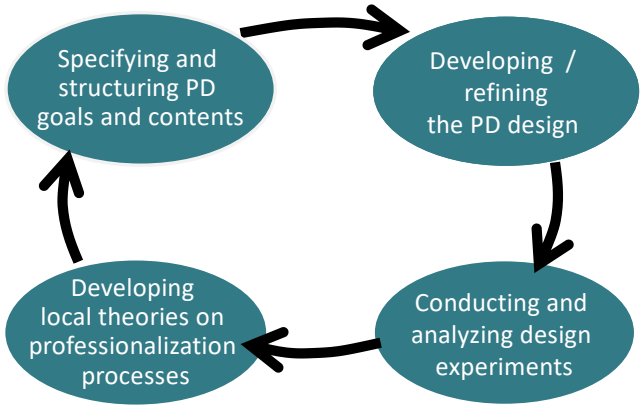
Design Experiment Cycle 1

Monitor students' learning

Enhance students' learning

Meanings of multiplication

3 x 5



Preparatory PD activities: (Brodie 2013; Karsenty 2010)

- introduce / revise basic concepts
- analyze student's ideas captured in given formative assessment tasks
- try and reflect the tasks and visuals in the Mastering Math materials

Teachers' experiments with students

PD settings

Design Experiment Cycle 1

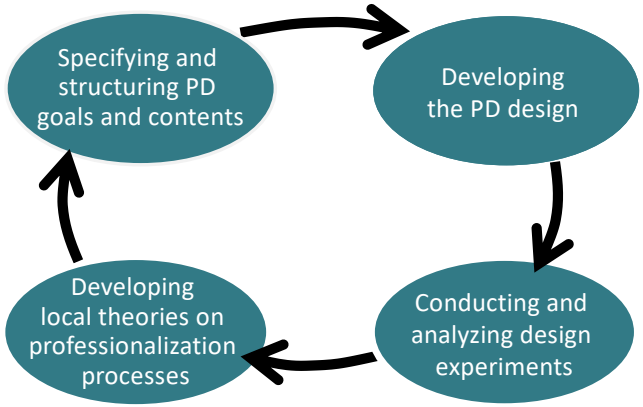
Monitor students' learning

Enhance students' learning

Meanings of multiplication

3 x 5

PCK categories about basic concepts



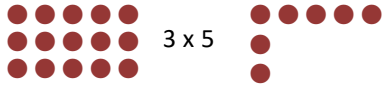
Teacher expertise for monitoring and enhance students' understanding requires PCK categories for deeply focusing mathematical structures

Preparatory PD activities:

- introduce / revise basic concepts
- analyze student's ideas captured in given formative assessment tasks
- try and reflect the tasks and visuals in the materials

Teachers like using dot arrays (as other visuals), but reflection about multiplication does not reach depth (Venkat & Askew 2018)

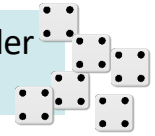
both correct?



unitizing structure rarely articulated

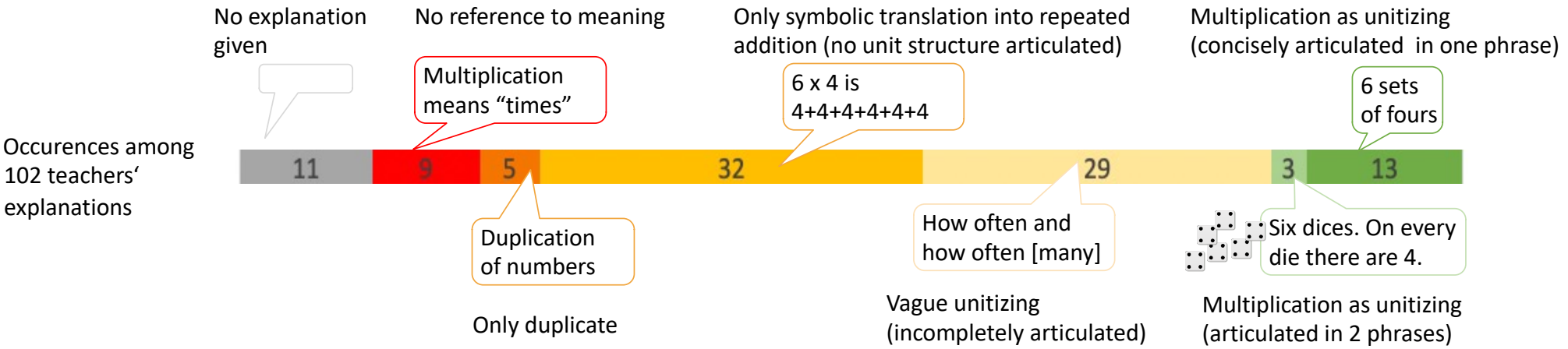
Teachers' PCK categories while explaining meanings

How would you explain to a fifth grader what multiplication means?



(later quantitative evidence in Cycle 5
for qualitative observations from Cycle 1-4)

Written answers of 102 highly motivated mathematics teachers volunteering at the beginning of an online PD in Cycle 5 (May/Sep 2022)



Summary: German mathematics teachers do a better job than the teachers observed by Mholo, Venkat, Schäfer (2012) in using and explaining the connection of representations
but still, we want them to be more explicit

(Prediger & Wischgoll, submitted, findings resonate with Askew, 2019; Barmby & Milinkovich, 2011)

Design Experiment Cycle 2

Specify learning content in basic concepts

Monitor students' learning in basic concepts

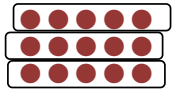
Enhance students' understanding in basic concepts

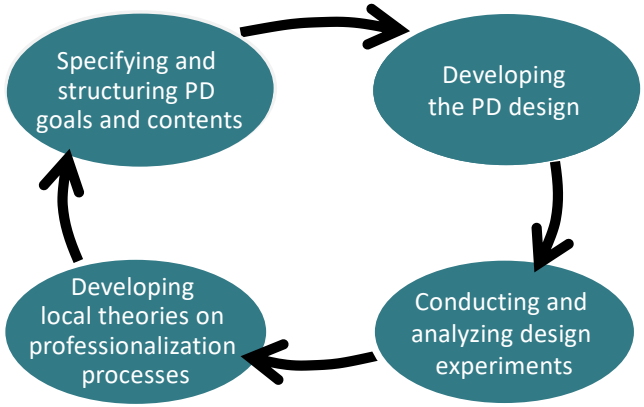
PCK categories about basic concepts

3 x 5

Meaning of multiplication

as counting in units



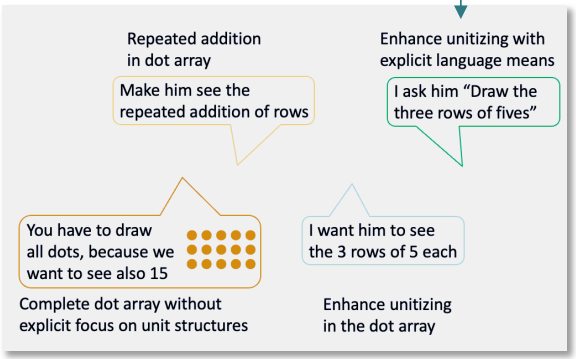


Preparatory PD activities to

- unpack basic concepts with their structures and articulations
- analyze student's ideas captured in given formative assessment tasks
- relate tasks and visuals to students' ideas

Torben has drawn this figure for explaining the meaning of 3 x 5.
How would you continue working with him?
How can the tasks in the material help?

Often not explicitly articulated



Teacher need stronger focus on mathematical structures

and more reflection about meaning-related language as an informal but concise language about structures

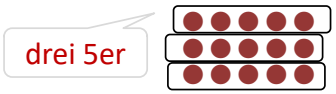
Design Experiment Cycle 3

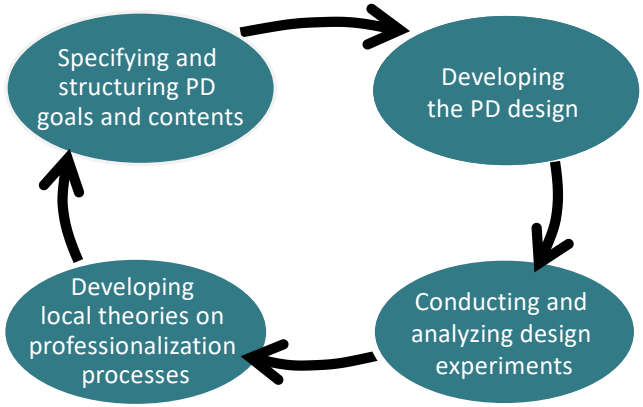
Specify learning content in basic concepts

Monitor students' learning in basic concepts

Enhance students' understanding in basic concepts

Meaning of multiplication as counting in units

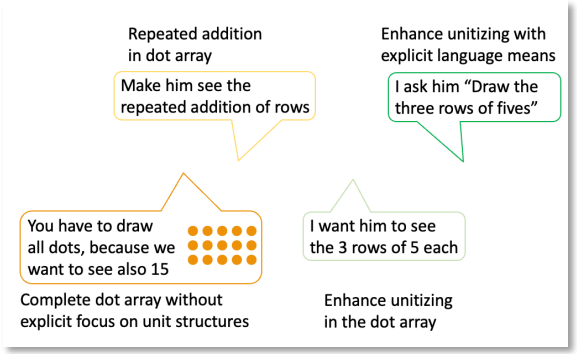




Preparatory PD activities to

- unpack basic concepts with their structures and articulations
- analyze student's ideas captured in given formative assessment tasks
- relate tasks and visuals to students' ideas and discuss relevant language means

Offered language was only partially adopted in teachers' enhancement practices



Teacher need stronger focus on mathematical structures

and more reflection about meaning-related language as an informal but concise language about structures

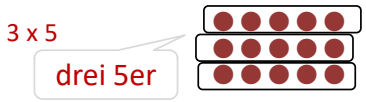
Design Experiment Cycle 4

Specify learning content in basic concepts

Monitor students' learning in basic concepts

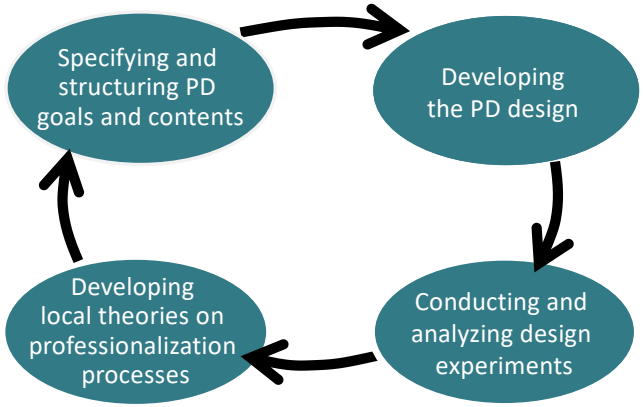
Enhance students' understanding in basic concepts

Meaning of multiplication as counting in units



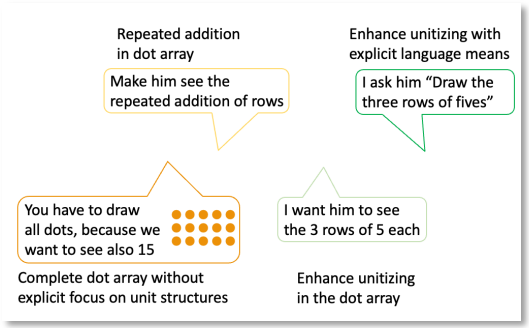
Preparatory PD activities to

- unpack basic concepts with their structures and articulations
- analyze student's ideas captured in given formative assessment tasks
- relate tasks and visuals to students' ideas
- discuss videos of enhancement practices also w.r.t. relevant language means



Teacher need stronger focus on mathematical structures

and more explicit video models for using meaning-related language

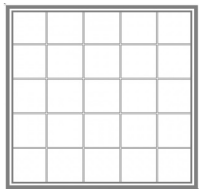


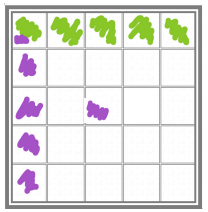
Design Experiment Cycle 4: Include very explicit video model

Tanya

Rayan

Task: Find an operation for this chocolate image





Rayan: 5 times 5

Teacher: Show me where you see the fives?

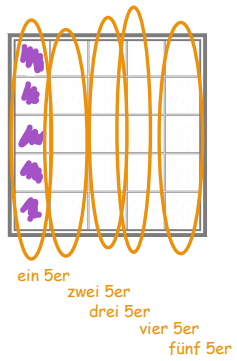
Rayan: Here

Teacher: But there is one field in both fives?

Rayan: Then, I must put it here

How would you react?

Teacher: Let us count the sets of fives



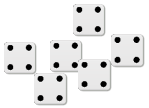
How does the teacher enhance students‘ deep understanding?

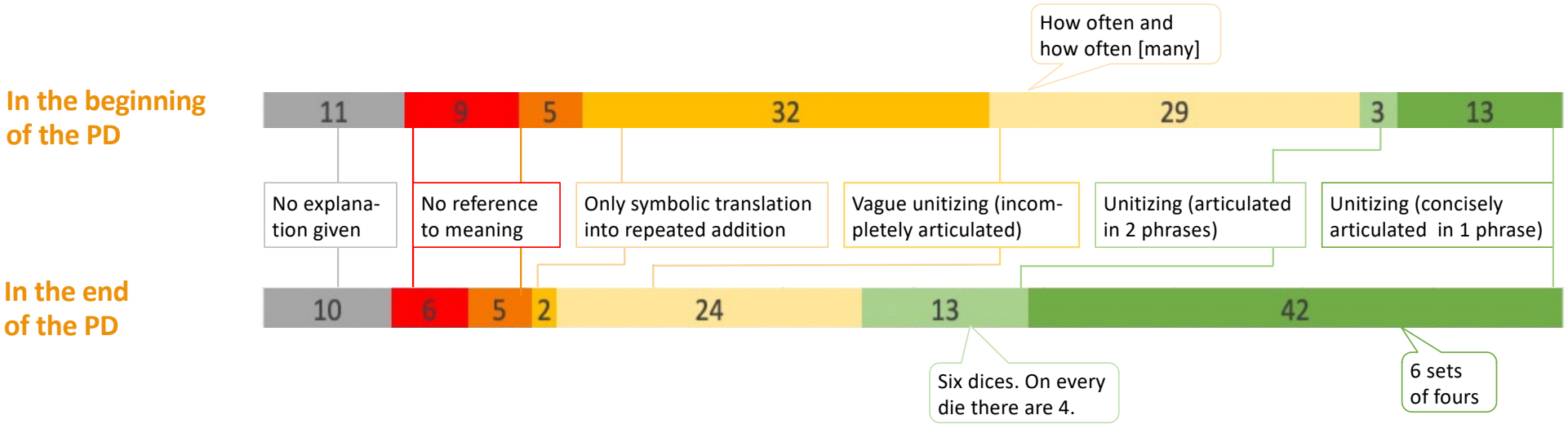
- supports mental unitizing in fives („5er“)
- meaning-related language supported by drawing and gestures as well as phrases

(Prediger 2019)

Changes in teachers' explanations of meanings (in Cycle 5)

How would teachers explain to a fifth grader what multiplication means?





(Prediger, Wischgoll & Rösike, in prep.)

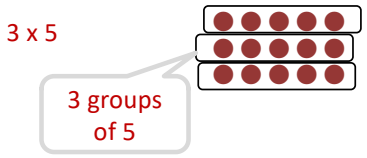
Design Experiment Cycle 5

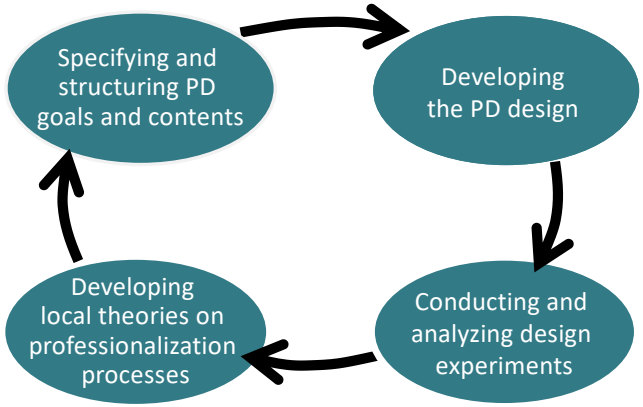
Specify learning content in basic concepts

Monitor students' learning in basic concepts

Enhance students' understanding in basic concepts

Meaning of multiplication as counting in units





Video cases can be highly valuable for deepening teachers' content-related and language-related experiences and changing their practices

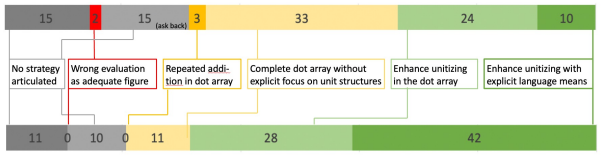
(Blomberg et al. 2013)

Preparatory PD activities to

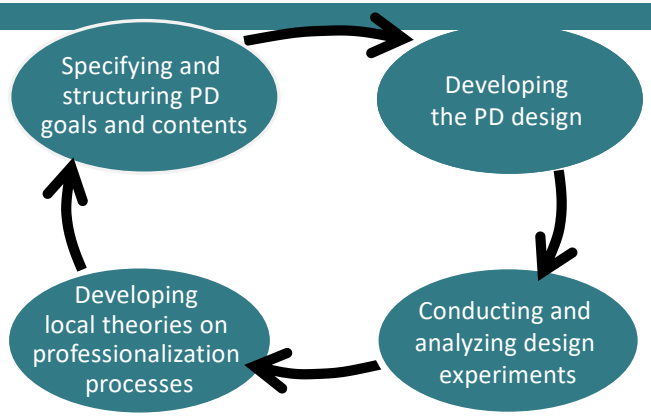
- unpack basic concepts with their structures and articulations
- analyze student's ideas captured in given formative assessment tasks
- relate tasks and visuals in the written enhancement materials to students' ideas

- discuss videos of enhancement practices also w.r.t. relevant language means

Offered language was substantially adopted



Main contribution of this project to generating theory



Unpacking the PD content in a model of content-related teacher expertise

Several practices for mastering the three jobs in different orientations

	Orientations	Specify learning content (in basic concepts)	Monitor students' learning progress (in basic concepts)	Enhance students' understanding (of basic concepts)
Compass	Diagnostic orientation OR	Adaptive goal-setting practice	(Takes monitoring seriously)	(Enhance with adaptive learning goal, see left)
	Syllabus-bound orientation	Syllabus-bound goal-setting practice	(Monitoring not necessary)	(Instruction of syllabus, see left)
Content	Conceptual orientation OR	Conceptual goal-setting practice	Conceptual diagnostic practices	Conceptual enhancement practice
	Procedural orientation	Procedural goal-setting practice	Procedural diagnostic practices	Procedural enhancement practice
Goal	Long-term orientation OR	Long-term foundation practices	Diagnostic practices with focus on foundations	Enhancement practices aiming at learning progress
	Short-term orientation	Short-term repair practices (focus current content)	Diagnostic practices with focus on current content	Compensation practices aiming at task completion
Pedagogy	Communicative orientation OR	(Required for conceptual learning content)	(Oral formative assessment in talk)	Communicative pedagogy
	Individualized orientation	(Realizable only for procedural learning content)	(Only written formative assessment)	Individualized pedagogy
Focus	Depth and correctness of addressed categories	Unpacking practices to varying degrees	Targeted diagnostic practices (with varying degrees of correctness)	(Targeted and adaptive enhancement practices)

Only after some PD design research cycles, the PD content is concisely specified,

so that we can construct good pre and post tests on teachers' PCK and practices for an evaluation study

Prediger, S. (in press). Using and developing content-related theory elements for explaining and promoting teachers' professional growth in collaborative groups. In H. Borko & D. Potari (Eds.), *Teachers of mathematics working and learning in collaborative groups. ICMI study*. Dordrecht: Springer.

Prediger, S., Dröse, J., Stahnke, R., & Ademmer, C. (2022, online first). Teacher expertise for fostering at-risk students' understanding of basic concepts: Conceptual model and evidence for growth. Journal of Mathematics Teacher Education. doi:10.1007/s10857-022-09538-3

Quantitative evidence for professional growth in a PD evaluation study (2019/20)

Teacher PD level

Research Question: Can we initiate professional growth by the Mastering Math PD program?

Sample
124 mathematics teachers
(no control group, yet)
in the Mastering Math PD program
over a year 6 session,
parallel to experiments with students

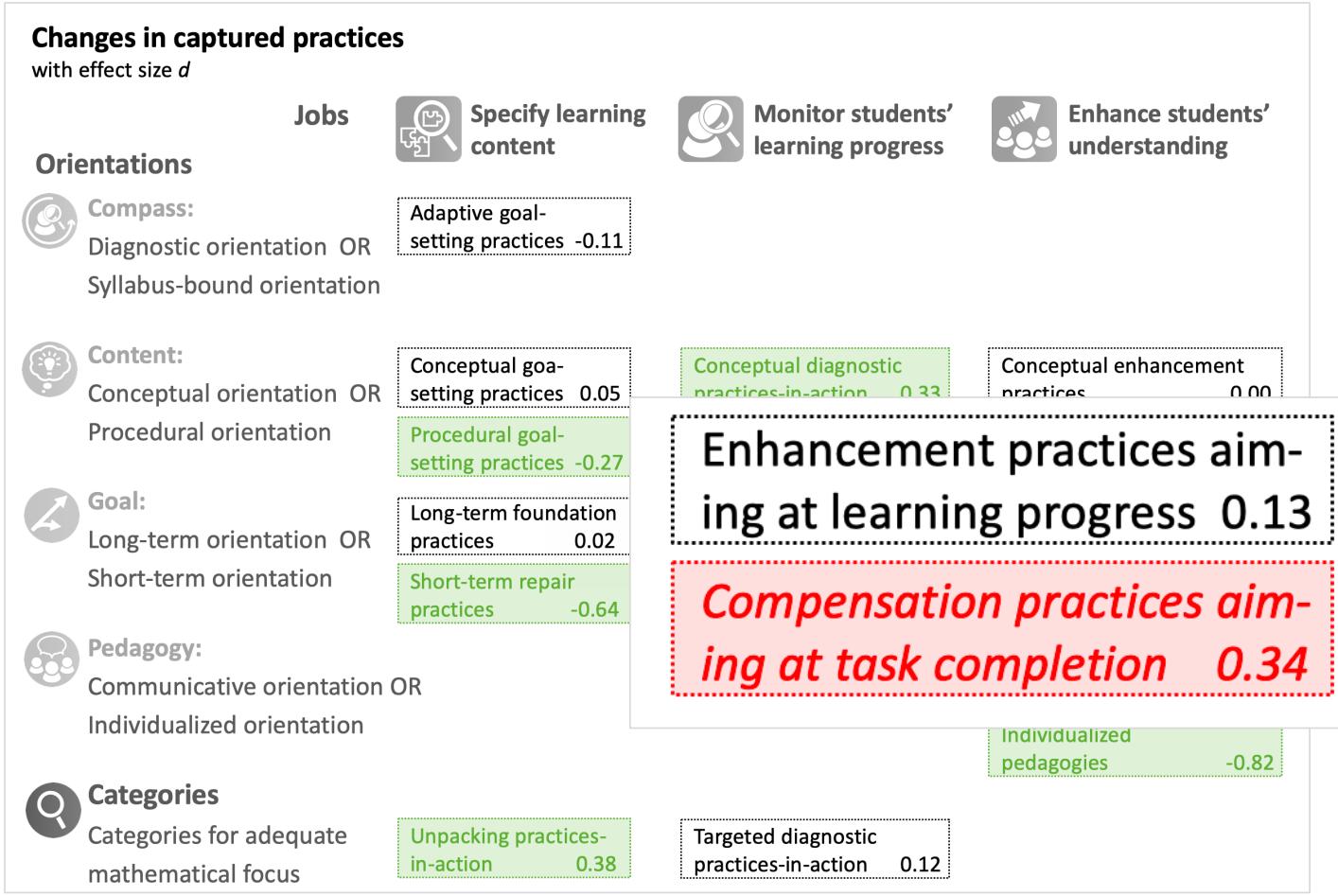
Results

Yes, partially

Significant growth in seven self-reported practices

but one challenge is left:
Overcoming short-term orientations

Changes in captured practices
with effect size *d*



(Prediger, Dröse, Stahnke, Ademmer 2022 JMTE)

Representations with long-term coherence

Number line

Coordinate system with function graphs

Negative numbers on the number line also <0

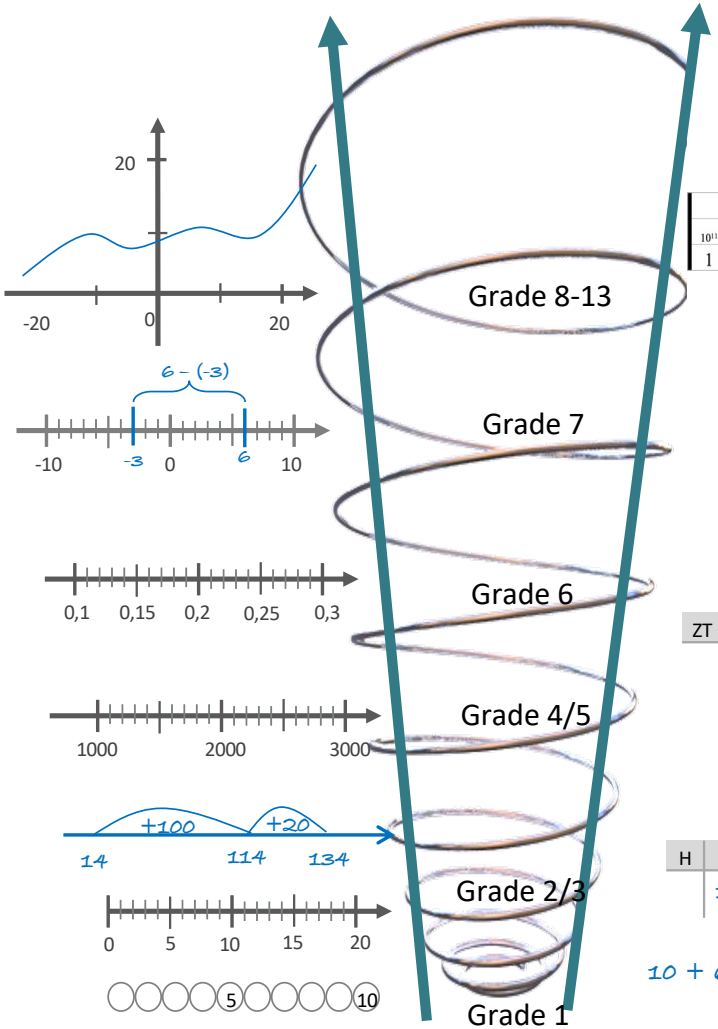
Decimal numbers zooming into the number line

Place value understanding zooming out of the number line

Concept-based calculation on the empty number line

Simply scaled number line

Lines of numbers



Place value table

Milliarden			Millionen			Tausend			Einer			Tausendstel		
10^{11}	10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10^{1}	10^{0}	10^{-1}	10^{-2}	10^{-4}
1	5	4	0	0	0	0	0	0	0	0	0			

Scientific notations with powers

km			m	dm	cm	mm
	1	6	0	2	3	5

160235 mm = 160,235 m

Converting unit measures

ZT	T	H	Z	E	z	h
	1	6	0	2,	3	5

Representing and calculating with decimal numbers in extended place value table

T	H	Z	E
x 10	1		6
1	6		

Calculating in the place value table

H	Z	E
	1	6

Representing numbers in the place value table

10 + 6

Understanding place values

Wittmann 1998 Standard representations

Prediger, Barzel, Hußmann, & Leuders (submitted). Durchgängigkeit von Darstellungen und Vorstellungen für den nachhaltigen Verstehensaufbau: Spiralcurriculum praktisch gewendet.

Representations with long-term coherence

Dot arrays and area models

Conditional probabilities
in the area model

Factoring expressions
in the area model

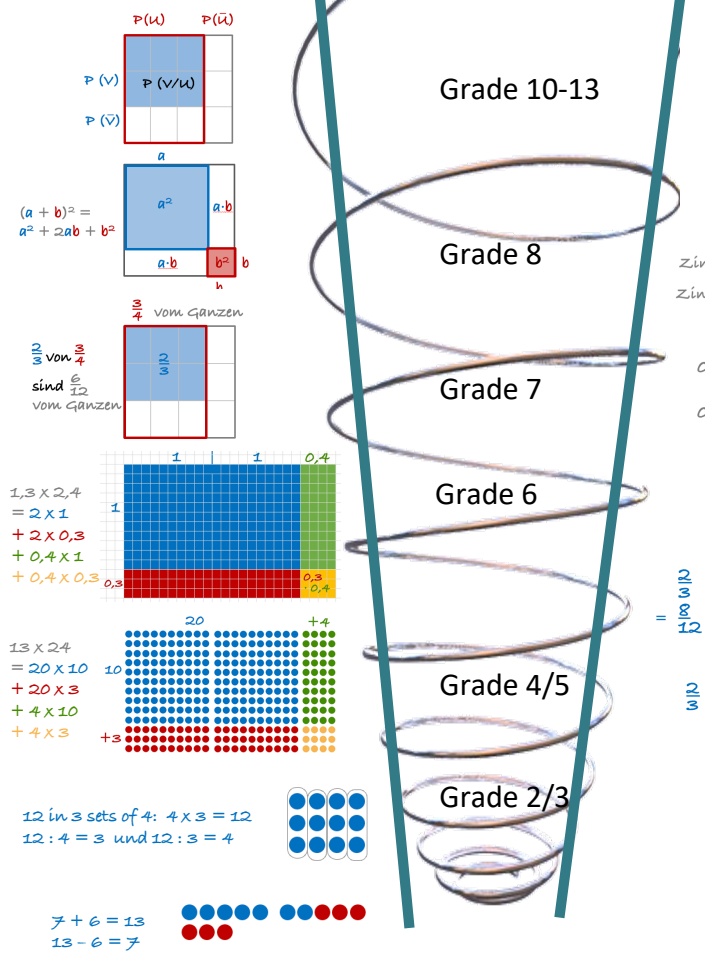
Meaning of multiplication
of fractions as part of part
in the area model

Meaning of multiplication
of decimals
in the area model

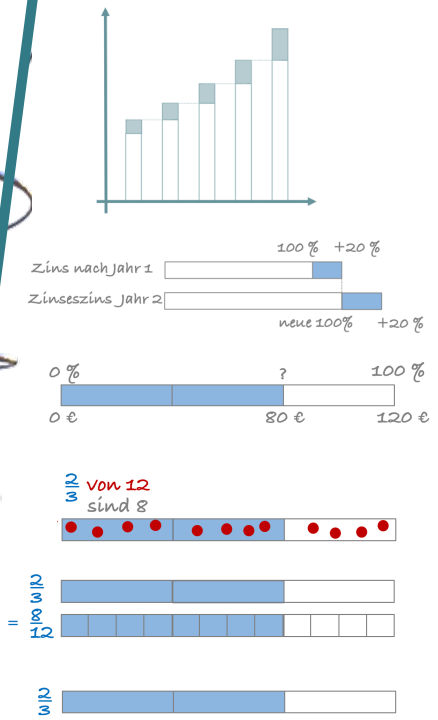
Double distributivity
in dot array

Meaning of multiplication
as counting in units
in dot arrays

Meaning of addition and
subtraction in array of 20



Bars



Exponential growth

Interest of interests

Determining percentages

Determining relative parts

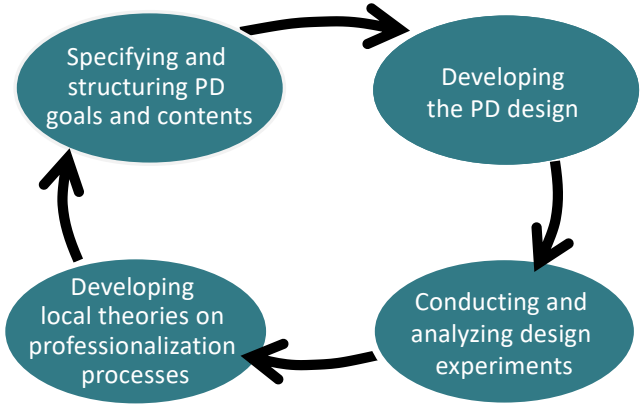
Comparing fractions

Representing fractions

(Wittmann 1998 Standard representations,
Prediger, Barzel, Hußmann, & Leuders submitted).

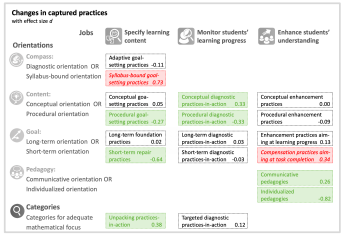
Consequences for the next design cycle

Longitudinal coherence
as a major PD content



Further work is needed
to consolidate these
first observations!

With the longitudinal as explicit PD content
teachers seem to better move to the
long-term orientation
(so far without new quantitative evidence)



Agenda for the talk

1. Two examples for introducing our challenges
2. Mastering Math – a design research project for fostering students' basic concepts in Grade 5-7
3. Mastering Math – a PD design research project for empowering teachers to foster students' basic concepts
4. Looking back on implementation strategies for bringing instructional innovations into classrooms

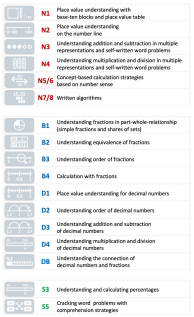
DZLM

Looking back onto a 16-year design research journey

Classroom level

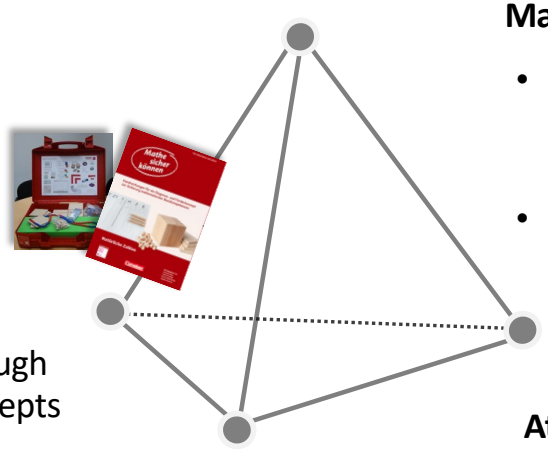
How can we as DZLM influence this unproductive instructional triangles?

By its fourth component!



Curriculum resources

- can support teachers in changing teaching practices
- can help to work through all relevant basic concepts

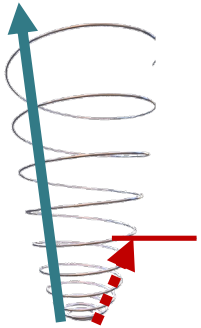


Mathematics teachers

- often use superficial teaching practices optimized for immediate task completion in short-term orientation
- should adopt long-term orientations

Mathematical content

- is highly cumulative with many longitudinal connections
- also procedures rely upon understanding basic concepts



At-risk students

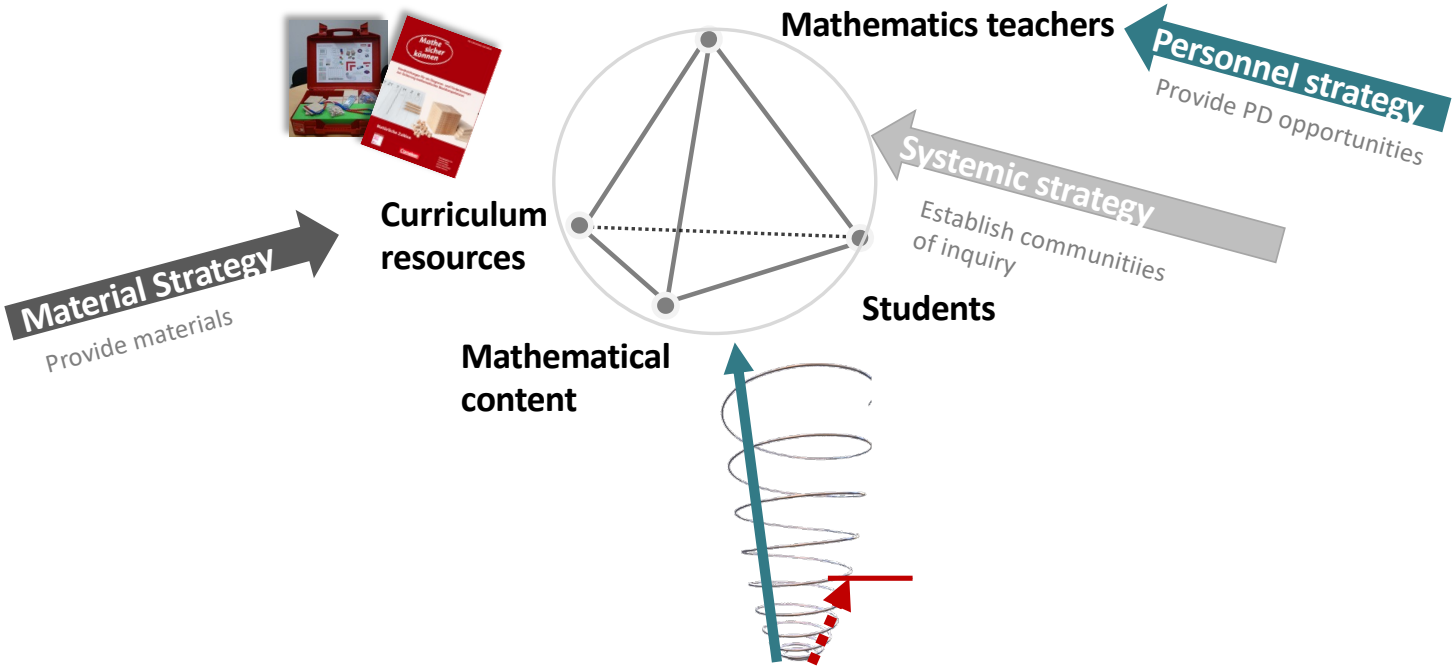
- often have insufficient learning opportunities for basic concepts
- cannot learn successfully in Grade 5-10 without understanding the basic concepts

How can we as DZLM support high-quality teaching and implement instructional innovations?

Curriculum resources alone do not do good teaching

Focus on mathematics teachers is crucial (Cohen, Raudenbush, Ball 2003; Desimone 2009)

Classroom level

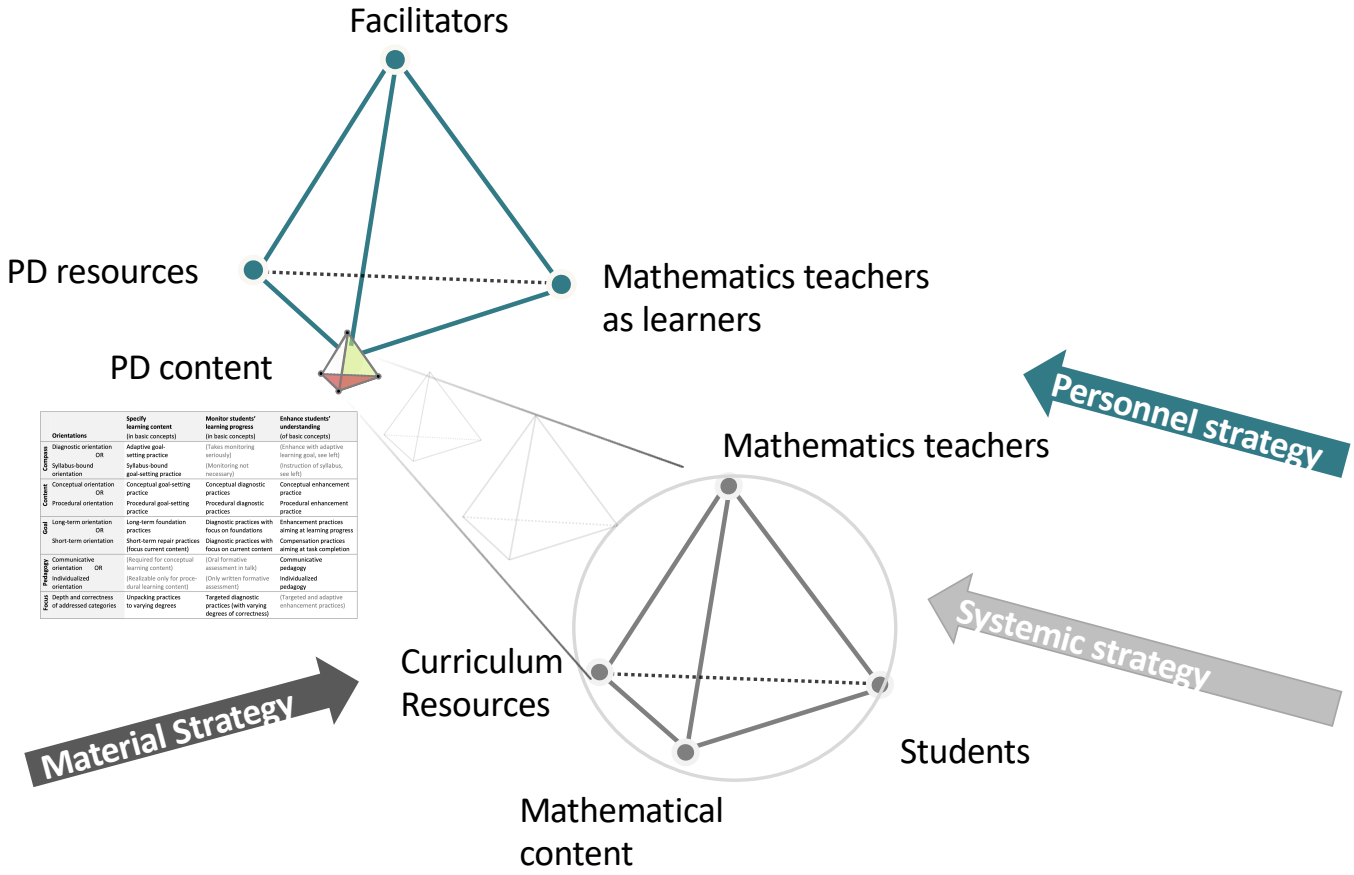


(Rösken-Winter, Prediger et al. 2021 ZDM)

How can we as DZLM support high-quality teaching and implement instructional innovations?

Teacher PD level

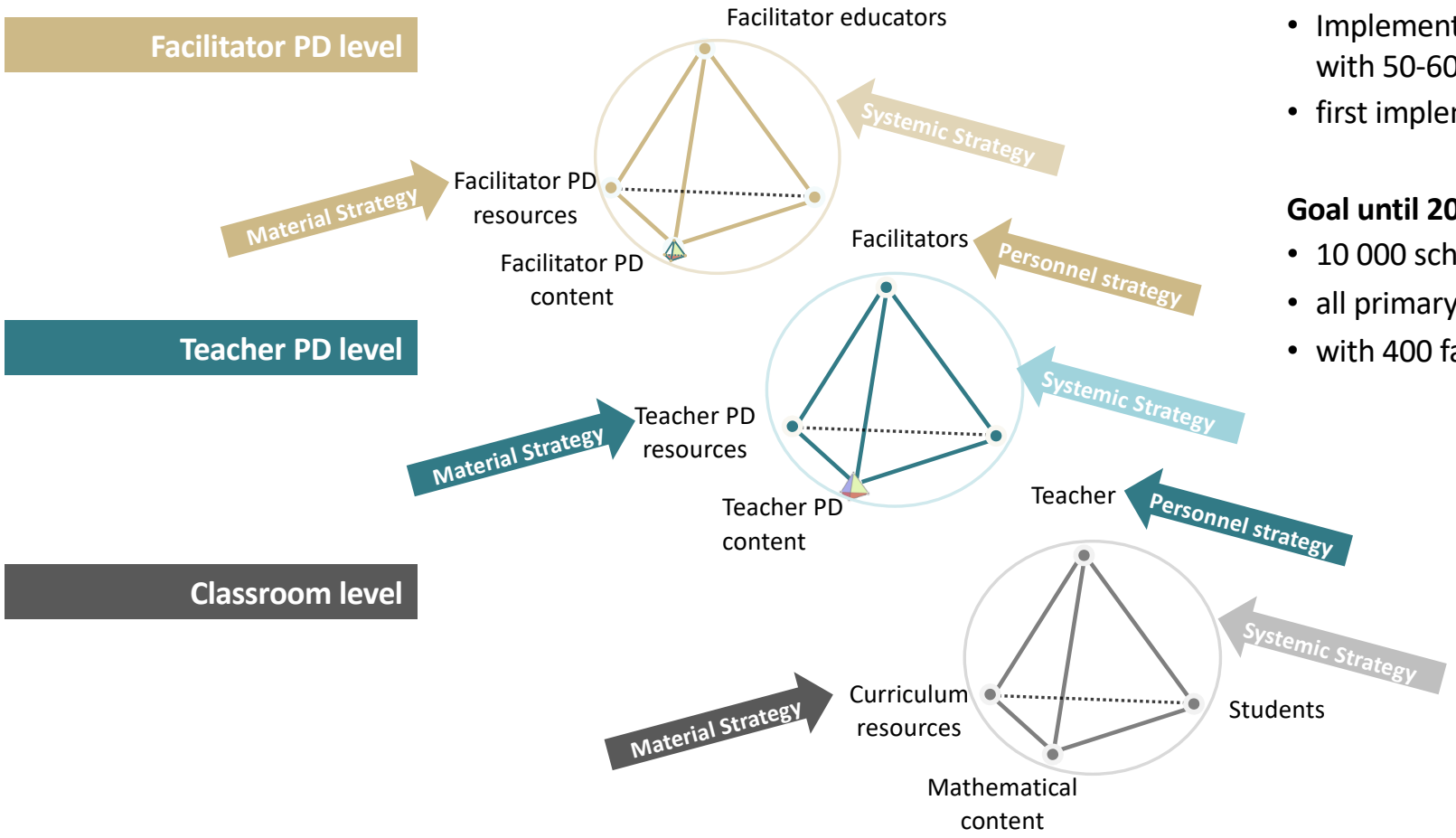
Classroom level



Orientations		Specify learning content (in basic concepts)	Monitor students' learning progress (in basic concepts)	Enhance students' understanding (of basic concepts)
Compass	Diagnostic orientation OR	Adaptive goal-setting practice	(Takes monitoring seriously)	(Enhance with adaptive learning goal, see left)
	Syllabus-bound orientation	Syllabus-bound goal-setting practice	(Monitoring not necessary)	(Instruction of syllabus, see left)
Content	Conceptual orientation OR	Conceptual goal-setting practice	Conceptual diagnostic practices	Conceptual enhancement practice
	Procedural orientation	Procedural goal-setting practice	Procedural diagnostic practices	Procedural enhancement practice
Goal	Long-term orientation OR	Long-term foundation practices	Diagnostic practices with focus on foundations	Enhancement practices aiming at learning progress
	Short-term orientation	Short-term repair practices (focus current content)	Diagnostic practices with focus on current content	Compensation practices aiming at task completion
Pedagogy	Communicative orientation OR	(Required for conceptual learning content)	(Oral formative assessment in talk)	Communicative pedagogy
	Individualized orientation	(Realizable only for procedural learning content)	(Only written formative assessment)	Individualized pedagogy
Focus	Depth and correctness of addressed categories	Unpacking practices to varying degrees	Targeted diagnostic practices (with varying degrees of correctness)	(Targeted and adaptive enhancement practices)

(Rösken-Winter, Prediger et al. 2021 ZDM)

Three implementation strategies in the 3 Tetrahedron Model



Current state of implementing Mastering Math:

- Implementation in 5 federal states with 50-60 schools each
- first implementation steps into 14 states

Goal until 2033:

- 10 000 schools until 2033
- all primary schools in some states
- with 400 facilitators nationwide

There is still a lot to investigate and design in the future

Thanks for your attention!

Roesken-Winter, B., Stahnke, R., Prediger, S., & Gasteiger, H. (2021). Towards a research base for implementation strategies addressing mathematics teachers and facilitators. *ZDM – Mathematics Education, 53*(5). http://doi.org/10.1007/s11858-021-01220-x

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Huge programs require great teams

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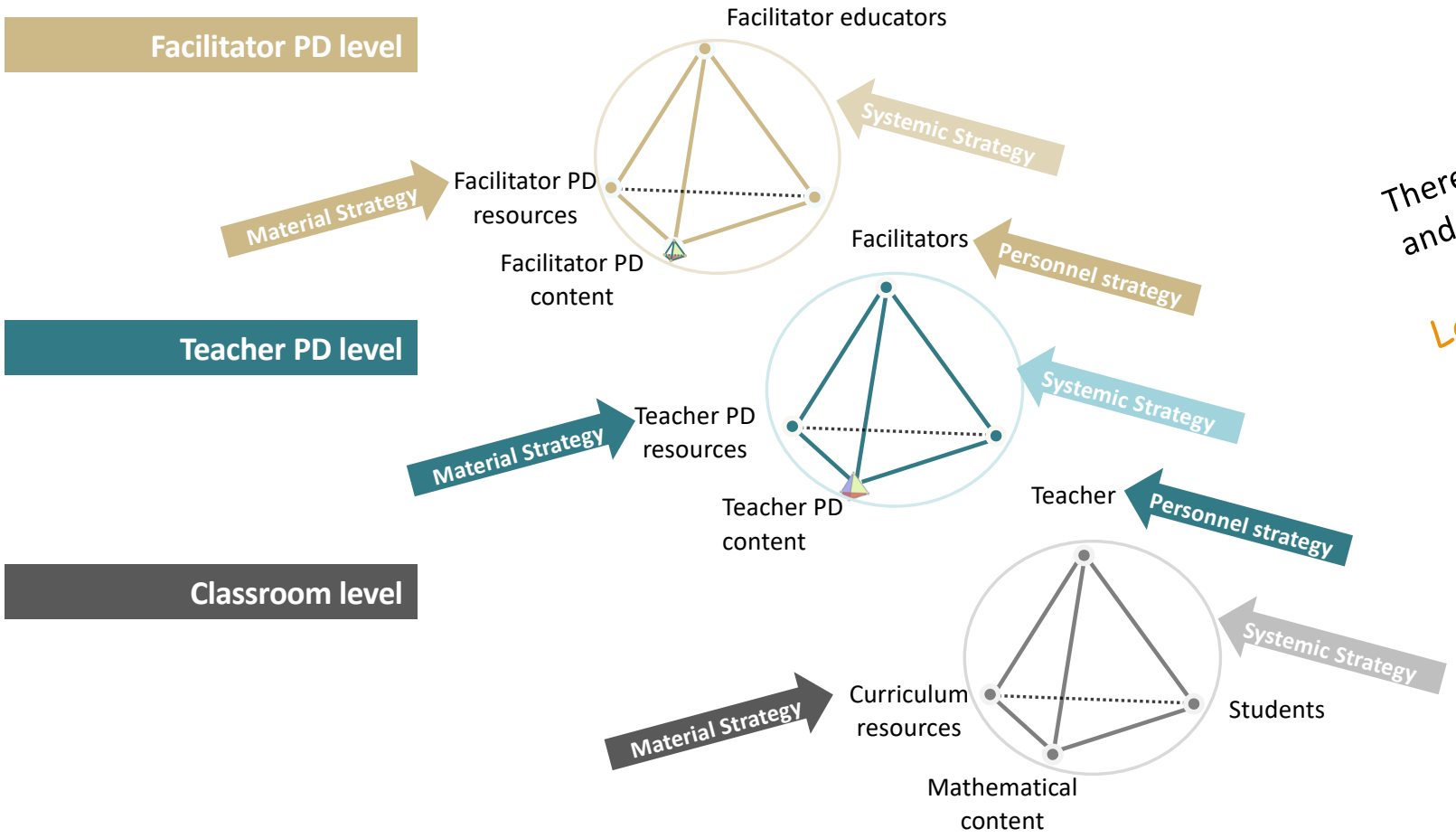
2 x N. N.

Three implementation strategies in the 3 Tetrahedron Model



There is still a lot to investigate and design in the future

Let us tackle it together!

Thanks for your attention!

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